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Supplement Materials for "High-Dimensional Volatility Matrix Estimation with Cross-Sectional Dependent and Heavy-Tailed Microstructural Noise"*

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The supplementary materials consist of four sections: A-D. To see the comparison among methods under different heavy tail situations, we consider generating the microstructure noise from the other two heavy tail distributions in Section A. In Section B, we perform sensitivity analyses of the rank r and sparsity thresholding ω_n. In Section C, we report additional results corresponding to when the noise is generated from the t-distribution. Finally, in Section D, we explain the reason for the absence of monotonicity of the average relative errors of the portfolio risks against p in Table 3 in the main text.

A Different heavy tail situations

In this Section, we consider generating the microstructure noise from the other two heavy tail distributions under the exponentially decaying correlation with $\rho = 0.5$. One is a log-normal distribution with parameters $\mu = 1$ and $\sigma = 1.2$, and the other is a Weibull distribution with shape parameter 0.3 and scale parameter 0.5. Tables A.1-A.5 report the results for $\|\widehat{\Gamma} - \Gamma\|_{\Gamma}$, average relative errors, $\|\widehat{\Gamma}^{-1} - \Gamma^{-1}\|_2$, $\|\widehat{\Sigma} - \Sigma\|_2$, and $\|\widehat{\Sigma}^{-1} - \Sigma^{-1}\|_2$, respectively.

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1) $\|\widehat{\Gamma} - \Gamma\|_{\Gamma}$ calculated under the exponentially decaying correlation and two heavy tail behaviors

Table A.1 Relative Frobenius norm errors under the exponentially decaying correlation and two heavy tail behaviors

		Original			PSD		
p	n	RPRVM	PRVM	CRPRVM	PRPRVM	PPRVM	PCRPRVM
				Log-normal			
52	200	1.271(1.634)	1.599(0.221)	**1.118**(0.973)	1.218(1.498)	1.524(0.197)	**1.101**(0.977)
	500	0.875(0.410)	1.330(0.157)	**0.757**(0.220)	0.842(0.349)	1.263(0.147)	**0.750**(0.223)
	1000	0.907(0.912)	1.175(0.191)	**0.744**(0.647)	0.866(0.849)	1.097(0.087)	**0.730**(0.645)
	2000	0.623(0.232)	0.978(0.098)	**0.523**(0.082)	0.583(0.113)	0.928(0.049)	**0.510**(0.042)
100	200	1.204(0.130)	2.208(0.106)	**1.179**(0.118)	1.195(0.127)	2.118(0.103)	**1.170**(0.111)
	500	0.932(0.399)	1.838(0.237)	**0.890**(0.323)	0.955(0.403)	1.750(0.237)	**0.899**(0.328)
	1000	0.818(0.423)	1.576(0.103)	**0.730**(0.204)	0.819(0.338)	1.492(0.091)	**0.735**(0.203)
	2000	0.761(0.873)	1.397(0.174)	**0.686**(0.602)	0.778(0.784)	1.309(0.164)	**0.684**(0.602)
200	200	1.480(0.139)	3.108(0.100)	**1.464**(0.117)	1.479(0.136)	3.018(0.090)	**1.466**(0.116)
	500	1.054(0.119)	2.554(0.089)	**1.040**(0.105)	1.101(0.136)	2.443(0.077)	**1.052**(0.097)
	1000	0.822(0.093)	2.219(0.087)	**0.812**(0.083)	0.914(0.113)	2.113(0.077)	**0.856**(0.087)
	2000	0.796(0.634)	1.994(0.632)	**0.751**(0.480)	0.796(0.206)	1.802(0.078)	**0.722**(0.170)
				Weibull			
52	200	1.107(0.329)	1.575(0.135)	**0.986**(0.164)	1.064(0.293)	1.483(0.100)	**0.973**(0.170)
	500	0.943(0.592)	1.292(0.129)	**0.784**(0.291)	0.921(0.538)	1.231(0.116)	**0.785**(0.293)
	1000	0.774(0.389)	1.126(0.082)	**0.630**(0.118)	0.756(0.322)	1.072(0.051)	**0.630**(0.117)
	2000	0.667(0.435)	0.997(0.268)	**0.569**(0.254)	0.645(0.321)	0.929(0.068)	**0.554**(0.226)
100	200	1.288(0.486)	2.194(0.180)	**1.238**(0.380)	1.282(0.481)	2.090(0.140)	**1.230**(0.387)
	500	0.892(0.143)	1.787(0.077)	**0.861**(0.103)	0.935(0.165)	1.700(0.065)	**0.871**(0.104)
	1000	0.808(0.618)	1.566(0.090)	**0.752**(0.380)	0.847(0.561)	1.479(0.067)	**0.764**(0.381)
	2000	0.870(1.281)	1.391(0.228)	**0.780**(1.020)	0.894(1.191)	1.295(0.157)	**0.782**(1.011)
200	200	1.818(1.281)	3.156(0.258)	**1.741**(1.100)	1.835(1.251)	3.057(0.235)	**1.758**(1.101)
	500	1.195(0.454)	2.599(0.248)	**1.132**(0.292)	1.246(0.404)	2.454(0.096)	**1.158**(0.295)
	1000	0.873(0.155)	2.226(0.098)	**0.859**(0.140)	0.976(0.190)	2.117(0.080)	**0.900**(0.147)
	2000	0.986(1.033)	1.948(0.200)	**0.954**(0.944)	1.092(1.028)	1.832(0.159)	**0.993**(0.940)

Note: This table presents the relative Frobenius norm errors of RPRVM-POET, PRVM, CRPRVM-POET, PRPRVM-POET, PPRVM, and PCRPRVM-POET estimators for Γ under the exponentially decaying correlation with $\rho = 0.5$ and two heavy tail behaviors, one being a log-normal distribution with parameters $\mu = 1$ and $\sigma = 1.2$, and the other being a Weibull distribution with shape parameter 0.3 and scale parameter 0.5.

2) Portfolio risks

Table A.2 Average relative errors under the exponentially decaying correlation and two heavy tail behaviors

		Original			PSD		
p	n	RPRVM	PRVM	CRPRVM	PRPRVM	PPRVM	PCRPRVM
				Log-normal			
52	200	0.399 (0.526)	0.341(0.140)	**0.334**(0.275)	0.397 (0.536)	**0.293**(0.131)	0.317 (0.283)
	500	0.322 (0.228)	0.275(0.098)	**0.221**(0.074)	0.333 (0.249)	0.246 (0.101)	**0.215**(0.079)
	1000	0.309 (0.285)	0.252(0.115)	**0.219**(0.107)	0.316 (0.300)	**0.210**(0.099)	0.213 (0.108)
	2000	0.278 (0.157)	0.211(0.083)	**0.184**(0.048)	0.282 (0.164)	0.195 (0.086)	**0.180**(0.050)
100	200	**0.266**(0.076)	0.334(0.115)	0.279 (0.070)	**0.240**(0.089)	0.268 (0.113)	0.246 (0.068)
	500	0.259 (0.147)	0.304(0.112)	**0.231**(0.057)	0.261 (0.161)	0.290 (0.157)	**0.222**(0.065)
	1000	0.219 (0.135)	0.273(0.122)	**0.193**(0.059)	0.215 (0.152)	0.230 (0.112)	**0.178**(0.063)
	2000	0.212 (0.208)	0.217(0.079)	**0.177**(0.114)	0.220 (0.231)	0.200 (0.090)	**0.168**(0.117)
200	200	0.261 (0.111)	0.334(0.125)	**0.255**(0.093)	0.247 (0.138)	0.309 (0.162)	**0.233**(0.109)
	500	**0.219**(0.074)	0.289(0.100)	0.220 (0.063)	0.207 (0.089)	0.310 (0.153)	**0.198**(0.064)
	1000	0.196 (0.082)	0.254(0.100)	**0.191**(0.071)	0.199 (0.098)	0.297 (0.168)	**0.181**(0.076)
	2000	**0.169**(0.068)	0.229(0.120)	0.172 (0.081)	0.159 (0.078)	0.239 (0.114)	**0.148**(0.054)
				Weibull			
52	200	0.345 (0.275)	0.307(0.097)	**0.278**(0.125)	0.353 (0.288)	**0.254**(0.092)	0.271 (0.136)
	500	0.322 (0.366)	0.255(0.125)	**0.232**(0.189)	0.330 (0.380)	0.230 (0.120)	**0.229**(0.192)
	1000	0.299 (0.226)	0.240(0.111)	**0.193**(0.063)	0.307 (0.235)	0.211 (0.067)	**0.190**(0.064)
	2000	0.255 (0.190)	0.189(0.099)	**0.158**(0.066)	0.259 (0.181)	0.167 (0.066)	**0.154**(0.060)
100	200	0.312 (0.156)	0.331(0.103)	**0.288**(0.095)	0.312 (0.172)	0.305 (0.130)	**0.275**(0.105)
	500	0.216 (0.064)	0.275(0.111)	**0.203**(0.052)	0.224 (0.085)	0.232 (0.085)	**0.196**(0.048)
	1000	0.220 (0.183)	0.244(0.115)	**0.182**(0.078)	0.229 (0.197)	0.210 (0.101)	**0.176**(0.083)
	2000	0.224 (0.306)	0.235(0.104)	**0.188**(0.177)	0.226 (0.338)	0.216 (0.122)	**0.180**(0.179)
200	200	0.293 (0.146)	0.344(0.140)	**0.274**(0.089)	0.281 (0.168)	0.344 (0.178)	**0.253**(0.101)
	500	0.245 (0.132)	0.294(0.124)	**0.218**(0.093)	0.258 (0.163)	0.325 (0.183)	**0.217**(0.106)
	1000	0.187 (0.061)	0.269(0.126)	**0.182**(0.045)	0.188 (0.089)	0.279 (0.179)	**0.172**(0.053)
	2000	0.194 (0.186)	0.213(0.100)	**0.164**(0.091)	0.208 (0.211)	0.224 (0.127)	**0.161**(0.107)

Note: This table presents the average relative errors of the portfolio risks calculated by the RPRVM-POET, PRVM, CRPRVM-POET, PRPRVM-POET, PPRVM, and PCRPRVM-POET estimators under the exponentially decaying correlation with $\rho = 0.5$ and two heavy tail behaviors, one being a log-normal distribution with parameters $\mu = 1$ and $\sigma = 1.2$, and the other being a Weibull distribution with shape parameter 0.3 and scale parameter 0.5.

3) $\|\widehat{\Gamma}^{-1} - \Gamma^{-1}\|_2$ calculated under the exponentially decaying correlation and two heavy tail behaviors

Table A.3 Spectral norm errors of the inverse integrated volatility matrix estimators under the exponentially decaying correlation and two heavy tail behaviors

		Original		PSD	
p	n	RPRVM	CRPRVM	PRPRVM	PCRPRVM
			Log-normal		
52	200	1.716 (0.855)	**1.158**(0.504)	0.540 (0.295)	**0.477**(0.117)
	500	0.957 (0.769)	**0.570**(0.299)	0.344 (0.086)	**0.331**(0.077)
	1000	0.984 (1.773)	**0.398**(0.175)	0.314 (0.189)	**0.269**(0.057)
	2000	0.771 (2.332)	**0.328**(0.177)	0.236 (0.049)	**0.221**(0.036)
100	200	3.071 (3.527)	**2.289**(2.718)	**0.381**(0.065)	0.392 (0.074)
	500	1.277 (1.329)	**0.886**(0.697)	**0.279**(0.063)	0.294 (0.065)
	1000	1.035 (2.032)	**0.518**(0.425)	**0.225**(0.038)	0.234 (0.050)
	2000	0.556 (0.501)	**0.384**(0.182)	0.217 (0.093)	**0.207**(0.042)
200	200	4.536 (3.771)	**3.587**(3.623)	**0.409**(0.087)	0.430 (0.094)
	500	1.440 (1.071)	**1.109**(1.024)	**0.262**(0.045)	0.275 (0.052)
	1000	**0.765**(0.675)	0.934 (1.200)	0.218 (0.021)	**0.215**(0.023)
	2000	0.659 (0.736)	**0.600**(0.600)	**0.268**(0.357)	0.355 (1.073)
			Weibull		
52	200	1.811 (1.578)	**1.096**(0.605)	0.512 (0.174)	**0.481**(0.152)
	500	0.728 (0.484)	**0.467**(0.213)	0.350 (0.101)	**0.317**(0.059)
	1000	0.906 (2.369)	**0.404**(0.337)	0.263 (0.058)	**0.255**(0.047)
	2000	0.385 (0.283)	**0.243**(0.056)	0.224 (0.048)	**0.213**(0.047)
100	200	2.200 (1.517)	**1.784**(1.353)	0.440 (0.107)	**0.433**(0.071)
	500	0.810 (0.447)	**0.644**(0.408)	**0.283**(0.062)	0.301 (0.068)
	1000	0.627 (0.976)	**0.439**(0.288)	**0.222**(0.030)	0.229 (0.035)
	2000	0.507 (0.665)	**0.334**(0.213)	0.210 (0.029)	**0.201**(0.025)
200	200	4.887 (2.965)	**2.967**(1.801)	**0.423**(0.090)	0.425 (0.086)
	500	1.297 (1.140)	**1.069**(0.996)	**0.267**(0.050)	0.275 (0.046)
	1000	**0.664**(0.936)	0.927 (1.925)	0.240 (0.030)	**0.233**(0.034)
	2000	**0.526**(0.816)	0.536 (1.205)	0.235 (0.026)	**0.220**(0.027)

Note: This table presents the spectral norm errors of the inverse integrated volatility matrix estimators calculated using the RPRVM-POET, CRPRVM-POET, PRPRVM-POET, and PCRPRVM-POET estimators under the exponentially decaying correlation with $\rho = 0.5$ and two heavy tail behaviors, one being a log-normal distribution with parameters $\mu = 1$ and $\sigma = 1.2$, and the other being a Weibull distribution with shape parameter 0.3 and scale parameter 0.5.

4) $\|\widehat{\Sigma} - \Sigma\|_2$ calculated under the exponentially decaying correlation and two heavy tail behaviors

Table A.4 Spectral norm errors of the sparse integrated volatility matrix estimators under the exponentially decaying correlation and two heavy tail behaviors

		Original		PSD	
p	n	RPRVM	CRPRVM	PRPRVM	PCRPRVM
			Log-normal		
52	200	8.662 (8.254)	**6.364** (3.042)	4.373 (1.444)	**4.177** (1.068)
	500	6.779 (6.635)	**4.707** (2.768)	3.915 (0.975)	**3.801** (1.297)
	1000	6.551 (7.325)	**4.969** (4.137)	4.243 (1.867)	**3.958** (1.997)
	2000	5.862 (6.044)	**4.012** (2.852)	3.450 (1.043)	**3.228** (0.684)
100	200	10.172 (5.439)	**8.986** (6.236)	6.153 (3.729)	**5.413** (3.441)
	500	7.253 (5.836)	**6.400** (4.305)	6.104 (3.632)	**5.295** (3.673)
	1000	8.750 (10.700)	**7.044** (6.425)	6.798 (4.585)	**6.173** (5.835)
	2000	7.498 (9.341)	**6.307** (6.118)	6.774 (5.044)	**4.877** (4.284)
200	200	17.088 (9.769)	**15.911**(10.356)	13.028 (10.369)	**11.646**(10.599)
	500	14.195 (11.570)	**13.253**(11.149)	12.291 (10.152)	**10.109** (9.172)
	1000	8.217 (8.556)	**7.402** (7.680)	11.127 (8.717)	**8.541** (7.680)
	2000	15.921 (39.246)	**13.986**(32.803)	12.418 (16.639)	**9.355** (14.916)
			Weibull		
52	200	7.857 (4.043)	**6.066** (3.669)	4.848 (2.224)	**4.749** (2.053)
	500	6.120 (4.636)	**4.469** (1.816)	**4.017** (1.089)	4.117 (1.606)
	1000	5.660 (4.986)	**4.134** (2.163)	3.910 (1.402)	**3.874** (2.048)
	2000	5.407 (8.440)	**4.460** (5.087)	4.018 (1.592)	**3.730** (1.691)
100	200	9.915 (6.194)	**8.887** (6.076)	7.409 (4.360)	**6.493** (3.915)
	500	**7.406** (5.518)	7.706 (5.738)	8.206 (5.648)	**7.296** (5.490)
	1000	7.215 (7.381)	**6.249** (4.328)	7.583 (4.796)	**5.908** (4.046)
	2000	10.496 (14.026)	**8.478** (8.871)	9.881 (8.381)	**7.783** (8.439)
200	200	23.094 (16.094)	**20.306**(16.148)	21.540 (16.482)	**20.361**(17.100)
	500	17.280 (17.434)	**16.902**(16.483)	17.529 (14.228)	**16.466**(16.557)
	1000	12.081 (9.940)	**11.662**(10.270)	14.916 (11.573)	**12.173**(10.433)
	2000	**12.169**(13.360)	14.324 (20.022)	**14.655**(13.572)	14.668 (20.071)

Note: This table presents the spectral norm errors of the sparse integrated volatility matrix estimators calculated from the POET procedures with the RPRVM, CRPRVM, PRPRVM, and PCRPRVM estimators under the exponentially decaying correlation with $\rho = 0.5$ and two heavy tail behaviors, one being a log-normal distribution with parameters $\mu = 1$ and $\sigma = 1.2$, and the other being a Weibull distribution with shape parameter 0.3 and scale parameter 0.5.

5) $\|\widehat{\Sigma}^{-1} - \Sigma^{-1}\|_2$ calculated under the exponentially decaying correlation and two heavy tail behaviors

Table A.5 Spectral norm errors of the inverse sparse integrated volatility matrix estimators under the exponentially decaying correlation and two heavy tail behaviors

		Original		PSD	
p	n	RPRVM	CRPRVM	PRPRVM	PCRPRVM
			Log-normal		
52	200	3.218 (4.624)	**1.848**(2.257)	0.643 (0.382)	**0.549**(0.186)
	500	6.185 (33.416)	**0.812**(0.965)	0.393 (0.114)	**0.364**(0.092)
	1000	0.726 (0.454)	**0.488**(0.340)	0.405 (0.383)	**0.329**(0.253)
	2000	0.567 (0.380)	**0.374**(0.271)	0.265 (0.068)	**0.239**(0.049)
100	200	12.423(45.383)	**3.991**(7.379)	**0.414**(0.090)	0.420 (0.101)
	500	3.066 (10.809)	**1.463**(3.470)	**0.301**(0.074)	0.313 (0.076)
	1000	3.768 (14.245)	**0.591**(0.586)	**0.242**(0.055)	0.248 (0.061)
	2000	0.755 (1.057)	**0.414**(0.215)	0.232 (0.139)	**0.214**(0.048)
200	200	19.856(89.167)	**5.507**(6.783)	**0.425**(0.095)	0.443 (0.102)
	500	8.811 (50.870)	**1.731**(4.701)	**0.270**(0.046)	0.284 (0.054)
	1000	**0.872** (0.924)	1.363 (3.169)	0.221 (0.021)	**0.218**(0.024)
	2000	0.716 (0.872)	**0.641**(0.689)	**0.271**(0.366)	0.368 (1.148)
			Weibull		
52	200	4.812 (7.990)	**1.956**(3.101)	0.606 (0.273)	**0.539**(0.218)
	500	1.425 (3.262)	**0.638**(0.704)	0.425 (0.199)	**0.353**(0.080)
	1000	0.789 (0.970)	**0.493**(0.514)	0.307 (0.104)	**0.282**(0.076)
	2000	0.589 (0.951)	**0.265**(0.069)	0.254 (0.070)	**0.233**(0.063)
100	200	11.183(45.537)	**3.706**(5.640)	0.476 (0.138)	**0.460**(0.093)
	500	1.139 (0.935)	**0.800**(0.637)	**0.302**(0.079)	0.321 (0.087)
	1000	0.760 (1.312)	**0.507**(0.463)	**0.235**(0.058)	0.238 (0.039)
	2000	1.048 (3.696)	**0.379**(0.305)	0.293 (0.568)	**0.215**(0.066)
200	200	9.292 (9.969)	**5.479**(7.361)	0.442 (0.101)	**0.439**(0.090)
	500	1.793 (2.240)	**1.626**(2.748)	**0.275**(0.053)	0.282 (0.048)
	1000	**0.723** (1.089)	1.955 (7.842)	0.242 (0.032)	**0.236**(0.037)
	2000	**0.612** (1.069)	0.704 (2.094)	0.237 (0.026)	**0.223**(0.028)

Note: This table presents the spectral norm errors of the inverse sparse integrated volatility matrix estimators, where the sparse integrated volatility matrix estimators are calculated from the POET procedures with the RPRVM, CRPRVM, PRPRVM, and PCRPRVM estimators under the exponentially decaying correlation with $\rho = 0.5$ and two heavy tail behaviors, one being a log-normal distribution with parameters $\mu = 1$ and $\sigma = 1.2$, and the other being a Weibull distribution with shape parameter 0.3 and scale parameter 0.5.

B Sensitivity analyses of the rank r and sparsity thresholding ω_n

In Section B, we perform sensitivity analyses of the rank r and sparsity thresholding ω_n. More specifically, for the sensitivity analysis of rank r, vary the number of common factors specified in the POET procedures to 2, 3, 4, 5, and 6 in the empirical study where the selected r is 4 and simulation scenario where the microstructure noise is generated from the t-distribution under the exponentially decaying correlation with $\rho = 0.5$, and the true number of common factors is 4. For the sensitivity analysis of sparsity thresholding ω_n, varying ω_n in the above simulation scenario and empirical study makes the sparsity level roughly 85%, 90%, 95%, and 100%, where the sparsity level is defined as the number of non-diagonal zero elements of $\widehat{\Sigma}$ divided by the total number of non-diagonal elements. The selected thresholding ω_n in both simulations and empirical study corresponds to a sparsity level of approximately 100%.

Subsections B.1 and B.2 show the results of the sensitivity analyses that are performed in the simulation study (Tables B.1-B.10) and the empirical study (Tables B.11-B.12), respectively. Further, in subsubsections B.1.1 and B.1.2, we show the results of sensitivity analysis for rank r

(Tables B.1-B.5) and sparsity thresholding ω_n (Tables B.6-B.10), respectively, where sensitivity analysis is performed in the simulation study.

B.1 Sensitivity analysis performed in the simulation study

B.1.1 Sensitivity analysis of rank r

1) $\|\widehat{\Gamma}-\Gamma\|_\Gamma$ calculated under the exponentially decaying correlation and different specified numbers of common factors

Table B.1 Relative Frobenius norm errors under the exponentially decaying correlation and different specified numbers of common factors

			Original			PSD		
p	n	r	RPRVM	PRVM	CRPRVM	PRPRVM	PPRVM	PCRPRVM
52	200	2	1.100 (0.295)	1.621 (0.213)	**1.066**(0.212)	1.151(0.196)	1.521 (0.142)	**1.105**(0.181)
		3	1.040 (0.339)	1.621 (0.213)	**0.982**(0.252)	1.013(0.217)	1.521 (0.142)	**0.964**(0.171)
		4	1.127 (0.364)	1.621 (0.213)	**1.020**(0.250)	1.048(0.262)	1.521 (0.142)	**0.970**(0.172)
		5	1.352 (0.317)	1.621 (0.213)	**1.219**(0.233)	1.247(0.229)	1.521 (0.142)	**1.171**(0.177)
		6	1.480 (0.302)	1.621 (0.213)	**1.338**(0.223)	1.371(0.218)	1.521 (0.142)	**1.292**(0.173)
	500	2	1.043 (0.134)	1.280 (0.068)	**1.039**(0.126)	1.115(0.137)	1.220 (0.061)	**1.075**(0.127)
		3	0.792 (0.113)	1.280 (0.068)	**0.776**(0.083)	0.828(0.108)	1.220 (0.061)	**0.792**(0.084)
		4	0.777 (0.175)	1.280 (0.068)	**0.714**(0.068)	0.754(0.143)	1.220 (0.061)	**0.707**(0.069)
		5	1.038 (0.161)	1.280 (0.068)	**0.890**(0.076)	0.994(0.136)	1.220 (0.061)	**0.880**(0.079)
		6	1.160 (0.160)	1.280 (0.068)	**0.999**(0.078)	1.109(0.138)	1.220 (0.061)	**0.989**(0.080)
	1000	2	1.128 (0.466)	1.134 (0.091)	**1.086**(0.268)	1.178(0.414)	**1.082**(0.082)	1.110 (0.269)
		3	0.826 (0.482)	1.134 (0.091)	**0.768**(0.258)	0.847(0.424)	1.082 (0.082)	**0.778**(0.258)
		4	0.738 (0.492)	1.134 (0.091)	**0.645**(0.251)	0.715(0.440)	1.082 (0.082)	**0.635**(0.250)
		5	1.049 (0.475)	1.134 (0.091)	**0.816**(0.274)	0.993(0.440)	1.082 (0.082)	**0.806**(0.273)
		6	1.170 (0.468)	1.134 (0.091)	**0.914**(0.264)	1.112(0.433)	1.082 (0.082)	**0.902**(0.264)
	2000	2	1.103 (0.171)	**0.979**(0.121)	1.084 (0.152)	1.141(0.143)	**0.922**(0.042)	1.097 (0.138)
		3	0.751 (0.158)	0.979 (0.121)	**0.704**(0.098)	0.751(0.089)	0.922 (0.042)	**0.702**(0.076)
		4	0.627 (0.226)	0.979 (0.121)	**0.529**(0.084)	0.575(0.137)	0.922 (0.042)	**0.508**(0.035)
		5	0.922 (0.197)	0.979 (0.121)	**0.666**(0.085)	0.858(0.145)	0.922 (0.042)	**0.647**(0.052)
		6	1.048 (0.194)	0.979 (0.121)	**0.754**(0.080)	0.983(0.145)	0.922 (0.042)	**0.736**(0.052)
100	200	2	**1.169**(0.168)	2.193 (0.099)	1.173 (0.158)	1.312(0.178)	2.108 (0.095)	**1.265**(0.167)
		3	1.101 (0.132)	2.193 (0.099)	**1.087**(0.113)	1.147(0.123)	2.108 (0.095)	**1.116**(0.113)
		4	1.210 (0.216)	2.193 (0.099)	**1.163**(0.139)	1.172(0.173)	2.108 (0.095)	**1.143**(0.130)
		5	1.505 (0.203)	2.193 (0.099)	**1.415**(0.148)	1.428(0.163)	2.108 (0.095)	**1.384**(0.128)
		6	1.687 (0.194)	2.193 (0.099)	**1.591**(0.135)	1.595(0.156)	2.108 (0.095)	**1.555**(0.117)
	500	2	1.269 (0.524)	1.856 (0.283)	**1.261**(0.495)	1.396(0.517)	1.758 (0.271)	**1.334**(0.491)
		3	1.023 (0.529)	1.856 (0.283)	**1.005**(0.497)	1.095(0.520)	1.758 (0.271)	**1.044**(0.492)
		4	0.969 (0.614)	1.856 (0.283)	**0.923**(0.539)	0.956(0.573)	1.758 (0.271)	**0.915**(0.538)
		5	1.276 (0.586)	1.856 (0.283)	**1.149**(0.545)	1.218(0.560)	1.758 (0.271)	**1.126**(0.540)
		6	1.433 (0.572)	1.856 (0.283)	**1.289**(0.531)	1.360(0.546)	1.758 (0.271)	**1.260**(0.526)
	1000	2	**1.232**(0.224)	1.607 (0.227)	1.233 (0.227)	1.365(0.230)	1.526 (0.230)	**1.303**(0.232)
		3	0.888 (0.233)	1.607 (0.227)	**0.879**(0.235)	0.983(0.236)	1.526 (0.230)	**0.925**(0.240)
		4	0.774 (0.295)	1.607 (0.227)	**0.731**(0.243)	0.790(0.292)	1.526 (0.230)	**0.734**(0.248)
		5	1.136 (0.303)	1.607 (0.227)	**0.937**(0.299)	1.079(0.286)	1.526 (0.230)	**0.931**(0.290)
		6	1.281 (0.290)	1.607 (0.227)	**1.059**(0.286)	1.213(0.272)	1.526 (0.230)	**1.048**(0.276)
	2000	2	1.386 (0.697)	1.412 (0.289)	**1.283**(0.280)	1.368(0.209)	1.314 (0.210)	**1.287**(0.190)
		3	0.998 (0.751)	1.412 (0.289)	**0.870**(0.285)	0.937(0.163)	1.314 (0.210)	**0.852**(0.119)
		4	0.795 (0.792)	1.412 (0.289)	**0.633**(0.301)	0.666(0.150)	1.314 (0.210)	**0.578**(0.050)
		5	1.169 (0.736)	1.412 (0.289)	**0.820**(0.285)	1.003(0.278)	1.314 (0.210)	**0.756**(0.085)
		6	1.310 (0.716)	1.412 (0.289)	**0.924**(0.272)	1.136(0.268)	1.314 (0.210)	**0.859**(0.081)
200	200	2	1.488 (0.627)	3.268 (1.253)	**1.430**(0.215)	1.602(0.305)	2.998 (0.092)	**1.521**(0.147)
		3	1.475 (0.617)	3.268 (1.253)	**1.405**(0.209)	1.485(0.329)	2.998 (0.092)	**1.418**(0.113)
		4	1.588 (0.604)	3.268 (1.253)	**1.500**(0.204)	1.506(0.388)	2.998 (0.092)	**1.449**(0.104)
		5	1.952 (0.566)	3.268 (1.253)	**1.835**(0.198)	1.828(0.368)	2.998 (0.092)	**1.768**(0.116)
		6	2.186 (0.546)	3.268 (1.253)	**2.058**(0.190)	2.048(0.352)	2.998 (0.092)	**1.986**(0.114)
	500	2	**1.318**(0.137)	2.534 (0.072)	1.320 (0.134)	1.513(0.141)	2.433 (0.063)	**1.442**(0.137)
		3	1.084 (0.102)	2.534 (0.072)	**1.079**(0.092)	1.203(0.092)	2.433 (0.063)	**1.146**(0.083)
		4	1.033 (0.087)	2.534 (0.072)	**1.018**(0.068)	1.066(0.067)	2.433 (0.063)	**1.027**(0.054)
		5	1.371 (0.131)	2.534 (0.072)	**1.276**(0.079)	1.337(0.102)	2.433 (0.063)	**1.266**(0.065)
		6	1.559 (0.132)	2.534 (0.072)	**1.450**(0.077)	1.510(0.108)	2.433 (0.063)	**1.436**(0.070)
	1000	2	1.324 (0.231)	2.232 (0.196)	**1.321**(0.233)	1.511(0.237)	2.126 (0.194)	**1.437**(0.232)
		3	1.002 (0.250)	2.232 (0.196)	**0.994**(0.247)	1.135(0.252)	2.126 (0.194)	**1.069**(0.244)
		4	0.880 (0.296)	2.232 (0.196)	**0.858**(0.260)	0.926(0.284)	2.126 (0.194)	**0.870**(0.255)
		5	1.207 (0.379)	2.232 (0.196)	**1.087**(0.317)	1.189(0.327)	2.126 (0.194)	**1.088**(0.296)
		6	1.381 (0.360)	2.232 (0.196)	**1.241**(0.302)	1.338(0.314)	2.126 (0.194)	**1.229**(0.284)
	2000	2	1.363 (0.353)	1.922 (0.112)	**1.323**(0.166)	1.534(0.214)	1.807 (0.059)	**1.435**(0.159)
		3	0.958 (0.381)	1.922 (0.112)	**0.909**(0.157)	1.090(0.212)	1.807 (0.059)	**0.993**(0.145)
		4	0.754 (0.413)	1.922 (0.112)	**0.686**(0.170)	0.808(0.248)	1.807 (0.059)	**0.715**(0.155)
		5	1.143 (0.482)	1.922 (0.112)	**0.914**(0.249)	1.110(0.381)	1.807 (0.059)	**0.915**(0.220)
		6	1.323 (0.466)	1.922 (0.112)	**1.042**(0.244)	1.252(0.368)	1.807 (0.059)	**1.033**(0.214)

Note: This table presents the relative Frobenius norm errors of RPRVM-POET, PRVM, CRPRVM-POET, PRPRVM-POET, PPRVM, and PCRPRVM-POET estimators for Γ under the exponentially decaying correlation with $\rho = 0.5$, varying the number of common factors specified in the POET procedures to 2, 3, 4, 5, and 6 for sensitivity analysis. The true number of common factors is 4.

2) Portfolio risks

Table B.2 Average relative errors under the exponentially decaying correlation and different specified numbers of common factors

			Original			PSD		
p	n	r	RPRVM	PRVM	CRPRVM	PRPRVM	PPRVM	PCRPRVM
52	200	2	0.306 (0.088)	0.350 (0.133)	**0.278**(0.068)	0.304(0.098)	0.302 (0.130)	**0.270**(0.072)
		3	0.342 (0.155)	0.347 (0.126)	**0.280**(0.070)	0.345(0.161)	0.305 (0.130)	**0.269**(0.072)
		4	0.385 (0.278)	0.345 (0.132)	**0.282**(0.095)	0.396(0.293)	0.302 (0.129)	**0.270**(0.099)
		5	0.497 (0.312)	0.347 (0.128)	**0.313**(0.126)	0.526(0.332)	0.303 (0.125)	**0.302**(0.136)
		6	0.542 (0.314)	0.342 (0.131)	**0.308**(0.129)	0.581(0.337)	**0.298**(0.126)	0.300 (0.139)
	500	2	0.244 (0.077)	0.261 (0.105)	**0.230**(0.067)	0.243(0.082)	0.225 (0.088)	**0.225**(0.067)
		3	0.249 (0.090)	0.268 (0.106)	**0.232**(0.062)	0.247(0.101)	0.231 (0.093)	**0.221**(0.064)
		4	0.250 (0.106)	0.261 (0.105)	**0.223**(0.061)	0.254(0.116)	0.225 (0.092)	**0.212**(0.061)
		5	0.505 (0.242)	0.265 (0.106)	**0.238**(0.087)	0.536(0.259)	**0.229**(0.094)	0.234 (0.093)
		6	0.622 (0.246)	0.264 (0.106)	**0.238**(0.096)	0.661(0.253)	**0.226**(0.091)	0.236 (0.100)
	1000	2	0.266 (0.141)	0.237 (0.096)	**0.231**(0.077)	0.271(0.148)	**0.203**(0.076)	0.229 (0.077)
		3	0.246 (0.157)	0.236 (0.095)	**0.209**(0.075)	0.250(0.170)	**0.202**(0.076)	0.204 (0.074)
		4	0.252 (0.174)	0.235 (0.098)	**0.199**(0.078)	0.259(0.185)	0.203 (0.078)	**0.190**(0.073)
		5	0.510 (0.269)	0.235 (0.098)	**0.212**(0.089)	0.531(0.279)	**0.201**(0.075)	0.207 (0.092)
		6	0.672 (0.283)	0.235 (0.092)	**0.219**(0.092)	0.696(0.289)	**0.200**(0.068)	0.215 (0.095)
	2000	2	0.237 (0.098)	0.227 (0.103)	**0.204**(0.064)	0.239(0.097)	**0.200**(0.088)	0.202 (0.062)
		3	0.231 (0.110)	0.226 (0.103)	**0.185**(0.065)	0.231(0.112)	0.200 (0.086)	**0.178**(0.061)
		4	0.240 (0.130)	0.226 (0.106)	**0.177**(0.060)	0.240(0.130)	0.197 (0.085)	**0.167**(0.052)
		5	0.645 (0.353)	0.226 (0.104)	**0.213**(0.100)	0.669(0.363)	**0.200**(0.091)	0.208 (0.104)
		6	0.805 (0.287)	0.229 (0.109)	**0.219**(0.110)	0.829(0.294)	**0.199**(0.082)	0.214 (0.111)
100	200	2	0.263 (0.071)	0.340 (0.137)	**0.258**(0.068)	0.258(0.084)	0.289 (0.124)	**0.245**(0.071)
		3	0.257 (0.068)	0.339 (0.125)	**0.251**(0.061)	0.254(0.087)	0.285 (0.119)	**0.234**(0.062)
		4	0.273 (0.099)	0.339 (0.130)	**0.257**(0.063)	0.270(0.126)	0.287 (0.120)	**0.237**(0.066)
		5	0.344 (0.176)	0.336 (0.128)	**0.273**(0.081)	0.371(0.210)	0.286 (0.118)	**0.264**(0.100)
		6	0.393 (0.212)	0.341 (0.136)	**0.279**(0.090)	0.426(0.245)	0.290 (0.130)	**0.269**(0.115)
	500	2	0.252 (0.083)	0.274 (0.093)	**0.238**(0.073)	0.256(0.092)	0.238 (0.103)	**0.232**(0.070)
		3	0.232 (0.088)	0.274 (0.093)	**0.223**(0.071)	0.230(0.105)	0.239 (0.107)	**0.208**(0.073)
		4	0.227 (0.098)	0.268 (0.090)	**0.217**(0.059)	0.222(0.119)	0.234 (0.099)	**0.196**(0.061)
		5	0.412 (0.241)	0.270 (0.092)	**0.235**(0.076)	0.450(0.273)	0.236 (0.103)	**0.226**(0.087)
		6	0.504 (0.262)	0.272 (0.092)	**0.229**(0.087)	0.551(0.286)	0.235 (0.106)	**0.231**(0.104)
	1000	2	0.237 (0.067)	**0.224**(0.077)	0.226 (0.063)	0.249(0.073)	**0.214**(0.086)	0.228 (0.063)
		3	0.201 (0.067)	0.227 (0.080)	**0.192**(0.056)	0.210(0.079)	0.214 (0.096)	**0.186**(0.057)
		4	0.197 (0.073)	0.226 (0.079)	**0.183**(0.066)	0.204(0.084)	0.212 (0.091)	**0.171**(0.064)
		5	0.409 (0.224)	0.221 (0.078)	**0.188**(0.072)	0.449(0.243)	0.212 (0.093)	**0.190**(0.081)
		6	0.567 (0.243)	0.228 (0.082)	**0.204**(0.080)	0.620(0.252)	0.218 (0.094)	**0.211**(0.092)
	2000	2	0.230 (0.064)	**0.212**(0.100)	0.213 (0.054)	0.228(0.050)	**0.189**(0.084)	0.207 (0.047)
		3	0.189 (0.069)	0.211 (0.098)	**0.172**(0.056)	0.183(0.053)	0.189 (0.081)	**0.160**(0.038)
		4	0.188 (0.101)	0.212 (0.096)	**0.167**(0.068)	0.176(0.067)	0.192 (0.081)	**0.150**(0.049)
		5	0.449 (0.265)	0.209 (0.094)	**0.183**(0.075)	0.478(0.278)	0.188 (0.075)	**0.176**(0.076)
		6	0.666 (0.297)	0.211 (0.097)	**0.189**(0.079)	0.701(0.297)	0.190 (0.079)	**0.187**(0.085)
200	200	2	0.302 (0.125)	0.388 (0.234)	**0.288**(0.072)	0.289(0.118)	0.346 (0.183)	**0.272**(0.075)
		3	0.298 (0.109)	0.382 (0.194)	**0.287**(0.068)	0.283(0.128)	0.357 (0.180)	**0.263**(0.074)
		4	0.279 (0.110)	0.378 (0.182)	**0.273**(0.075)	0.266(0.128)	0.349 (0.184)	**0.246**(0.075)
		5	0.289 (0.134)	0.383 (0.223)	**0.270**(0.076)	0.305(0.161)	0.351 (0.182)	**0.251**(0.073)
		6	0.348 (0.187)	0.391 (0.222)	**0.283**(0.097)	0.377(0.213)	0.349 (0.181)	**0.276**(0.111)
	500	2	0.254 (0.069)	0.286 (0.115)	**0.252**(0.067)	0.259(0.077)	0.301 (0.174)	**0.249**(0.068)
		3	0.226 (0.073)	0.279 (0.111)	**0.225**(0.074)	0.225(0.073)	0.302 (0.172)	**0.214**(0.070)
		4	**0.203**(0.060)	0.280 (0.112)	0.210 (0.056)	0.196(0.074)	0.303 (0.171)	**0.189**(0.058)
		5	0.307 (0.178)	0.280 (0.111)	**0.223**(0.079)	0.351(0.218)	0.299 (0.168)	**0.218**(0.099)
		6	0.390 (0.263)	0.278 (0.111)	**0.232**(0.098)	0.446(0.300)	0.297 (0.173)	**0.239**(0.124)
	1000	2	0.229 (0.085)	0.237 (0.088)	**0.222**(0.076)	0.245(0.101)	0.281 (0.137)	**0.227**(0.085)
		3	0.217 (0.086)	0.237 (0.088)	**0.209**(0.077)	0.224(0.107)	0.284 (0.137)	**0.205**(0.087)
		4	0.196 (0.075)	0.238 (0.093)	**0.194**(0.068)	0.195(0.095)	0.284 (0.145)	**0.180**(0.075)
		5	0.302 (0.189)	0.241 (0.093)	**0.201**(0.083)	0.343(0.225)	0.290 (0.153)	**0.200**(0.098)
		6	0.401 (0.229)	0.237 (0.090)	**0.206**(0.085)	0.462(0.255)	0.287 (0.143)	**0.216**(0.103)
	2000	2	0.219 (0.071)	0.218 (0.077)	**0.205**(0.054)	0.246(0.063)	0.260 (0.105)	**0.220**(0.051)
		3	0.177 (0.064)	0.218 (0.080)	**0.164**(0.042)	0.190(0.068)	0.258 (0.099)	**0.163**(0.044)
		4	0.170 (0.077)	0.221 (0.077)	**0.156**(0.045)	0.162(0.082)	0.259 (0.109)	**0.139**(0.047)
		5	0.328 (0.204)	0.217 (0.076)	**0.160**(0.053)	0.388(0.233)	0.255 (0.105)	**0.168**(0.070)
		6	0.488 (0.272)	0.218 (0.079)	**0.175**(0.063)	0.558(0.290)	0.257 (0.103)	**0.194**(0.081)

Note: This table presents the average relative errors of the portfolio risks calculated by the RPRVM-POET, PRVM, CRPRVM-POET, PRPRVM-POET, PPRVM, and PCRPRVM-POET estimators under the exponentially decaying correlation with $\rho = 0.5$, varying the number of common factors specified in the POET procedures to 2, 3, 4, 5, and 6 for sensitivity analysis. The true number of common factors is 4.

3) $\|\widehat{\Gamma}^{-1} - \Gamma^{-1}\|_2$ calculated under the exponentially decaying correlation and different specified numbers of common factors

Table B.3 Spectral norm errors of the inverse integrated volatility matrix estimators under the exponentially decaying correlation and different specified numbers of common factors

			Original		PSD	
p	n	r	RPRVM	CRPRVM	PRPRVM	PCRPRVM
52	200	2	0.969 (1.337)	**0.657** (0.543)	**0.274**(0.052)	0.280 (0.051)
		3	1.280 (0.820)	**1.193** (1.512)	**0.377**(0.105)	0.383 (0.095)
		4	2.566 (2.419)	**1.561** (0.960)	0.509 (0.124)	**0.487**(0.110)
		5	3.006 (1.593)	**2.066** (1.413)	0.699 (0.174)	**0.619**(0.158)
		6	**4.249** (2.919)	18.666(111.769)	0.918 (0.264)	**0.777**(0.214)
	500	2	0.348 (0.193)	**0.285** (0.066)	**0.258**(0.023)	0.261 (0.029)
		3	0.510 (0.287)	**0.380** (0.160)	**0.254**(0.051)	0.262 (0.055)
		4	0.891 (0.631)	**0.588** (0.273)	0.347 (0.072)	**0.338**(0.067)
		5	1.805 (2.253)	**0.751** (0.383)	0.491 (0.127)	**0.396**(0.073)
		6	1.975 (1.289)	**0.886** (0.354)	0.617 (0.154)	**0.451**(0.079)
	1000	2	0.440 (1.218)	**0.271** (0.052)	**0.257**(0.018)	0.257 (0.021)
		3	0.505 (1.132)	**0.324** (0.214)	0.236 (0.039)	**0.234**(0.027)
		4	0.570 (0.311)	**0.452** (0.340)	0.289 (0.128)	**0.269**(0.049)
		5	1.583 (3.333)	**0.618** (0.573)	0.467 (0.209)	**0.313**(0.075)
		6	2.122 (3.592)	**0.761** (0.813)	0.641 (0.213)	**0.386**(0.134)
	2000	2	0.378 (0.396)	**0.360** (0.364)	0.263 (0.016)	**0.262**(0.017)
		3	0.408 (0.467)	**0.367** (0.470)	**0.226**(0.020)	0.229 (0.025)
		4	0.574 (0.488)	**0.366** (0.245)	0.238 (0.067)	**0.233**(0.044)
		5	1.027 (0.699)	**0.444** (0.238)	0.431 (0.188)	**0.283**(0.122)
		6	1.961 (2.469)	**0.532** (0.273)	0.609 (0.245)	**0.338**(0.140)
100	200	2	1.324 (2.037)	**1.237** (3.361)	**0.282**(0.047)	0.288 (0.055)
		3	2.581 (5.695)	**1.188** (0.728)	**0.314**(0.064)	0.340 (0.076)
		4	2.759 (2.069)	**2.435** (3.282)	**0.396**(0.077)	0.427 (0.086)
		5	3.489 (1.568)	**2.865** (1.778)	**0.477**(0.094)	0.517 (0.134)
		6	5.447 (3.433)	**3.660** (3.148)	**0.552**(0.099)	0.598 (0.128)
	500	2	0.561 (0.702)	**0.429** (0.376)	0.279 (0.021)	**0.278**(0.023)
		3	0.859 (0.649)	**0.852** (1.093)	**0.247**(0.025)	0.255 (0.035)
		4	1.153 (0.879)	**1.125** (1.203)	**0.261**(0.045)	0.282 (0.056)
		5	2.055 (2.081)	**1.266** (0.863)	0.328 (0.082)	**0.325**(0.059)
		6	2.449 (2.185)	**1.659** (1.767)	0.416 (0.229)	**0.366**(0.068)
	1000	2	0.460 (0.769)	**0.349** (0.208)	0.278 (0.015)	**0.277**(0.015)
		3	0.644 (0.967)	**0.506** (0.552)	0.246 (0.026)	**0.241**(0.021)
		4	0.980 (1.383)	**0.592** (0.498)	**0.224**(0.040)	0.232 (0.032)
		5	1.490 (2.143)	**0.757** (0.516)	0.278 (0.065)	**0.263**(0.053)
		6	2.810 (5.111)	**0.964** (0.612)	0.362 (0.093)	**0.292**(0.058)
	2000	2	0.624 (1.147)	**0.467** (0.538)	**0.286**(0.018)	0.290 (0.048)
		3	**0.406** (0.331)	0.638 (1.488)	**0.250**(0.020)	0.254 (0.054)
		4	1.985 (10.541)	**1.002** (2.931)	**0.201**(0.025)	0.225 (0.197)
		5	2.077 (6.047)	**0.612** (0.513)	**0.323**(0.283)	0.368 (0.843)
		6	2.431 (3.737)	**0.703** (0.558)	0.409 (0.351)	**0.407**(0.894)
200	200	2	1.837 (2.784)	**1.497** (2.121)	0.286 (0.022)	**0.285**(0.022)
		3	3.180 (3.889)	**3.019** (4.059)	**0.305**(0.054)	0.312 (0.049)
		4	5.507 (4.711)	**4.115** (3.785)	**0.382**(0.082)	0.383 (0.070)
		5	**5.259** (3.217)	5.368 (5.856)	0.455 (0.204)	**0.435**(0.084)
		6	6.758 (4.034)	**4.939** (2.928)	0.509 (0.212)	**0.506**(0.092)
	500	2	**0.731** (1.129)	0.765 (1.012)	0.292 (0.016)	**0.291**(0.016)
		3	**1.131** (1.840)	1.244 (2.398)	**0.263**(0.022)	0.263 (0.031)
		4	1.559 (1.160)	**1.521** (1.443)	**0.268**(0.057)	0.281 (0.055)
		5	2.093 (1.312)	**1.589** (0.977)	**0.286**(0.069)	0.302 (0.056)
		6	3.332 (2.924)	**2.124** (1.651)	**0.309**(0.068)	0.331 (0.059)
	1000	2	**0.664** (1.236)	0.810 (2.198)	0.294 (0.016)	**0.293**(0.016)
		3	0.844 (1.968)	**0.816** (1.660)	0.265 (0.019)	**0.263**(0.020)
		4	0.918 (0.606)	**0.786** (0.620)	**0.220**(0.026)	0.220 (0.033)
		5	1.857 (2.550)	**1.234** (1.175)	**0.222**(0.029)	0.238 (0.032)
		6	2.115 (2.125)	**1.187** (0.686)	**0.236**(0.041)	0.249 (0.031)
	2000	2	**0.389** (0.390)	0.498 (0.871)	0.297 (0.016)	**0.295**(0.015)
		3	**0.389** (0.348)	0.409 (0.447)	0.267 (0.020)	**0.263**(0.019)
		4	0.720 (1.295)	**0.541** (0.697)	0.220 (0.029)	**0.204**(0.030)
		5	1.474 (1.712)	**0.803** (1.012)	0.227 (0.100)	**0.217**(0.013)
		6	1.491 (0.988)	**0.750** (0.523)	0.237 (0.116)	**0.226**(0.023)

Note: This table presents the spectral norm errors of the inverse integrated volatility matrix estimators calculated using the RPRVM-POET, CRPRVM-POET, PRPRVM-POET, and PCRPRVM-POET estimators under the exponentially decaying correlation with $\rho = 0.5$, varying the number of common factors specified in the POET procedures to 2, 3, 4, 5, and 6 for sensitivity analysis. The true number of common factors is 4.

4) $\|\widehat{\Sigma} - \Sigma\|_2$ calculated under the exponentially decaying correlation and different specified numbers of common factors

Table B.4 Spectral norm errors of the sparse integrated volatility matrix estimators under the exponentially decaying correlation and different specified numbers of common factors

			Original		PSD	
p	n	r	RPRVM	CRPRVM	PRPRVM	PCRPRVM
52	200	2	11.629 (8.374)	**10.242** (4.635)	10.659 (3.848)	**10.037** (3.115)
		3	9.291 (7.720)	**7.653** (6.256)	6.173 (2.367)	**5.648** (2.029)
		4	9.235 (6.850)	**7.967** (6.389)	4.355 (0.951)	**4.183** (1.016)
		5	10.528 (5.560)	**8.412** (5.351)	4.532 (0.547)	**4.448** (0.705)
		6	10.321 (5.861)	**8.842** (5.281)	4.878 (0.518)	**4.651** (0.586)
	500	2	10.370 (3.373)	**10.361** (2.896)	10.816 (2.862)	**10.500** (2.758)
		3	**6.414** (3.255)	6.444 (2.884)	6.607 (2.526)	**6.447** (2.595)
		4	5.894 (3.890)	**4.692** (2.514)	**3.721** (0.820)	3.755 (1.498)
		5	7.429 (2.788)	**5.089** (2.387)	4.097 (0.370)	**3.756** (0.242)
		6	7.702 (2.130)	**5.316** (2.288)	4.405 (0.299)	**3.990** (0.204)
	1000	2	11.888 (4.510)	**11.248** (3.093)	11.442 (3.340)	**11.168** (3.198)
		3	7.763 (4.362)	**7.211** (2.930)	7.235 (2.845)	**7.002** (2.759)
		4	5.735 (4.436)	**4.391** (2.707)	3.877 (1.843)	**3.588** (1.577)
		5	7.947 (3.328)	**4.590** (2.346)	4.002 (0.410)	**3.525** (0.220)
		6	7.802 (2.761)	**5.109** (2.479)	4.361 (0.294)	**3.792** (0.242)
	2000	2	12.104 (4.105)	**11.773** (3.488)	12.006 (3.101)	**11.610** (2.834)
		3	8.526 (5.153)	**7.603** (3.563)	7.569 (2.594)	**7.075** (2.184)
		4	6.268 (5.722)	**4.620** (3.757)	3.499 (1.011)	**3.207** (0.906)
		5	7.874 (4.028)	**4.848** (3.349)	3.944 (0.411)	**3.341** (0.285)
		6	7.994 (3.420)	**5.069** (3.238)	4.301 (0.292)	**3.570** (0.263)
100	200	2	12.744 (5.406)	**12.557** (4.876)	13.893 (5.612)	**13.122** (4.955)
		3	9.849 (6.090)	**9.413** (5.309)	8.645 (5.077)	**7.946** (4.365)
		4	11.349 (7.568)	**9.889** (5.829)	5.582 (3.483)	**5.255** (3.362)
		5	13.168 (4.361)	**10.545** (5.460)	4.344 (1.036)	**4.130** (0.590)
		6	13.014 (3.694)	**11.454** (4.418)	4.311 (0.509)	**4.284** (0.449)
	500	2	**15.106** (7.380)	15.265 (7.182)	15.183 (3.850)	**14.727** (4.336)
		3	10.849 (7.974)	**10.552** (7.317)	10.307 (3.862)	**9.741** (3.487)
		4	9.203 (9.118)	**6.983** (7.319)	5.242 (2.544)	**4.348** (1.720)
		5	11.868 (6.871)	**8.661** (7.519)	3.681 (0.457)	**3.576** (0.423)
		6	12.426 (6.110)	**9.530** (7.392)	3.903 (0.453)	**3.683** (0.342)
	1000	2	15.569 (6.453)	**15.545** (6.617)	16.246 (6.883)	**15.764** (6.946)
		3	11.075 (7.444)	**10.980** (7.269)	11.815 (7.432)	**11.015** (7.407)
		4	7.341 (6.082)	**7.016** (7.022)	6.275 (3.884)	**5.597** (6.751)
		5	10.816 (5.295)	**6.444** (4.194)	3.710 (1.107)	**3.250** (0.488)
		6	11.086 (3.721)	**6.994** (4.390)	3.745 (0.323)	**3.317** (0.326)
	2000	2	21.887 (28.538)	**18.886**(13.719)	16.926 (5.592)	**15.980** (5.293)
		3	17.234 (28.953)	**12.885**(13.879)	11.387 (4.416)	**9.993** (3.447)
		4	13.447 (29.746)	**7.799** (14.569)	6.438 (4.684)	**3.919** (2.301)
		5	16.531 (28.480)	**8.830** (13.506)	3.648 (0.894)	**3.144** (0.574)
		6	16.686 (28.294)	**9.201** (13.579)	3.798 (0.568)	**3.344** (0.572)
200	200	2	25.428 (34.091)	**22.426**(18.044)	19.275(10.993)	**17.519** (9.917)
		3	24.154 (29.959)	**20.596**(18.005)	15.032 (9.683)	**13.441** (8.860)
		4	22.469 (20.878)	**19.728**(16.405)	10.563 (8.709)	**9.102** (8.410)
		5	23.861 (19.342)	**20.478**(14.088)	5.536 (1.984)	**5.247** (2.089)
		6	23.746 (19.051)	**21.422**(12.994)	4.591 (0.889)	**4.522** (0.937)
	500	2	19.907 (8.713)	**19.428** (7.990)	18.523 (5.528)	**17.398** (4.990)
		3	15.889 (9.745)	**15.202** (8.765)	13.195 (4.819)	**11.937** (3.647)
		4	11.834 (8.041)	**10.732** (7.354)	8.752 (4.664)	**6.813** (3.481)
		5	16.616 (7.202)	**12.077** (7.931)	5.200 (1.382)	**4.428** (1.069)
		6	16.636 (5.590)	**12.955** (7.367)	4.132 (1.109)	**3.748** (0.620)
	1000	2	22.145 (22.682)	**21.703**(22.075)	22.613(23.395)	**21.274**(22.044)
		3	17.187 (23.152)	**16.625**(22.772)	16.774(23.865)	**15.294**(22.782)
		4	11.035 (11.669)	**10.545**(11.207)	9.913 (7.756)	**7.894** (9.095)
		5	13.801 (9.365)	**9.428** (7.574)	4.972 (1.851)	**4.021** (1.463)
		6	15.758 (7.075)	**11.391** (7.262)	3.824 (1.176)	**3.425** (0.713)
	2000	2	22.858 (21.244)	**20.275**(10.978)	22.146(12.523)	**20.252** (9.600)
		3	17.158 (21.117)	**14.947**(11.250)	16.439(11.120)	**14.281** (8.900)
		4	12.866 (21.687)	**9.870** (12.107)	11.815(11.712)	**8.501** (9.892)
		5	15.179 (11.655)	**8.535** (9.208)	5.604 (2.530)	**4.178** (1.880)
		6	16.932 (10.167)	**9.726** (8.426)	3.991 (1.224)	**3.147** (0.523)

Note: This table presents the spectral norm errors of the sparse integrated volatility matrix estimators calculated from the POET procedures with the RPRVM, CRPRVM, PRPRVM, and PCRPRVM estimators under the exponentially decaying correlation with $\rho = 0.5$, varying the number of common factors specified in the POET procedures to 2, 3, 4, 5, and 6 for sensitivity analysis. The true number of common factors is 4.

5) $\|\widehat{\Sigma}^{-1} - \Sigma^{-1}\|_2$ calculated under the exponentially decaying correlation and different specified numbers of common factors

Table B.5 Spectral norm errors of the inverse sparse integrated volatility matrix estimators under the exponentially decaying correlation and different specified numbers of common factors

			Original				PSD	
p	n	r	RPRVM		CRPRVM		PRPRVM	PCRPRVM
52	200	2	1.157	(1.468)	**0.847**	(0.965)	**0.301**(0.094)	0.302 (0.074)
		3	**2.257**	(3.551)	3.315	(11.845)	0.430 (0.155)	**0.425**(0.126)
		4	4.055	(4.469)	**2.686**	(3.037)	0.605 (0.221)	**0.555**(0.153)
		5	5.747	(4.252)	**3.047**	(2.531)	0.988 (0.810)	**0.883**(0.746)
		6	8.569	(8.333)	**6.189**	(6.783)	1.310 (0.896)	**1.218**(1.420)
	500	2	0.419	(0.326)	**0.299**	(0.077)	**0.270**(0.029)	0.272 (0.035)
		3	0.628	(0.437)	**0.422**	(0.211)	**0.274**(0.083)	0.277 (0.080)
		4	1.590	(2.398)	**0.774**	(0.614)	0.424 (0.149)	**0.381**(0.096)
		5	6.549	(28.331)	**1.222**	(1.300)	0.641 (0.326)	**0.466**(0.129)
		6	4.396	(4.725)	**1.451**	(1.150)	0.831 (0.412)	**0.538**(0.143)
	1000	2	0.320	(0.201)	**0.291**	(0.071)	0.274 (0.039)	**0.270**(0.025)
		3	0.435	(0.332)	**0.392**	(0.445)	0.260 (0.102)	**0.246**(0.041)
		4	0.716	(0.484)	**0.622**	(0.897)	0.415 (0.712)	**0.295**(0.060)
		5	4.302	(17.184)	**1.242**	(2.757)	0.724 (0.931)	**0.366**(0.108)
		6	1693.377	(11929.354)	**0.849**	(0.781)	0.986 (0.920)	**0.500**(0.281)
	2000	2	0.435	(0.599)	**0.402**	(0.533)	0.273 (0.019)	**0.272**(0.020)
		3	0.464	(0.527)	**0.418**	(0.583)	**0.237**(0.025)	0.239 (0.031)
		4	0.666	(0.558)	**0.411**	(0.299)	0.297 (0.264)	**0.250**(0.050)
		5	1.958	(2.469)	**0.540**	(0.411)	0.687 (0.649)	**0.326**(0.210)
		6	5.212	(10.363)	**0.804**	(0.884)	1.046 (1.059)	**0.410**(0.248)
100	200	2	2.470	(5.290)	**1.569**	(3.924)	**0.291**(0.055)	0.297 (0.060)
		3	5.439	(15.385)	**1.649**	(1.599)	**0.326**(0.074)	0.352 (0.087)
		4	**6.000**	(9.598)	6.775	(24.582)	**0.422**(0.091)	0.452 (0.097)
		5	6.619	(7.703)	**5.500**	(7.360)	**0.539**(0.177)	0.571 (0.192)
		6	11.434	(14.182)	**10.084**	(31.821)	0.672 (0.370)	**0.667**(0.174)
	500	2	2.849	(15.121)	**0.461**	(0.420)	**0.284**(0.022)	0.284 (0.024)
		3	**1.103**	(1.053)	1.552	(4.472)	**0.256**(0.030)	0.264 (0.040)
		4	**1.469**	(1.314)	2.269	(5.900)	**0.277**(0.056)	0.293 (0.063)
		5	8.171	(28.024)	**2.124**	(2.835)	0.435 (0.494)	**0.431**(0.570)
		6	6.786	(16.459)	**2.555**	(3.145)	0.675 (1.411)	**0.510**(0.802)
	1000	2	0.496	(0.920)	**0.361**	(0.211)	0.283 (0.016)	**0.281**(0.016)
		3	0.759	(1.164)	**0.604**	(0.886)	0.256 (0.039)	**0.252**(0.027)
		4	1.294	(1.925)	**0.653**	(0.566)	**0.236**(0.053)	0.243 (0.040)
		5	1.666	(1.424)	**0.998**	(0.858)	0.339 (0.132)	**0.332**(0.273)
		6	3.309	(3.676)	**1.363**	(1.214)	0.490 (0.235)	**0.372**(0.297)
	2000	2	0.851	(1.720)	**0.592**	(0.937)	**0.292**(0.019)	0.296 (0.049)
		3	**0.483**	(0.594)	0.953	(2.601)	**0.256**(0.021)	0.262 (0.056)
		4	**0.649**	(0.814)	1.581	(4.923)	**0.209**(0.027)	0.237 (0.213)
		5	4.870	(17.302)	**0.962**	(1.633)	1.216 (4.439)	**0.837**(2.871)
		6	3.947	(5.862)	**3.223**	(16.765)	1.454 (4.888)	**0.905**(2.917)
200	200	2	**2.917**	(5.797)	4.054	(16.778)	0.290 (0.023)	**0.290**(0.023)
		3	10.887	(46.880)	**6.962**	(19.077)	**0.315**(0.061)	0.319 (0.053)
		4	10.626	(20.556)	**7.414**	(10.214)	0.397 (0.093)	**0.393**(0.074)
		5	**7.641**	(5.087)	14.187	(40.132)	0.588 (0.932)	**0.450**(0.091)
		6	11.650	(12.556)	**8.407**	(6.787)	0.687 (1.204)	**0.535**(0.121)
	500	2	2.473	(13.018)	**1.229**	(3.199)	0.296 (0.018)	**0.294**(0.017)
		3	**1.729**	(5.431)	4.723	(25.786)	**0.267**(0.025)	0.268 (0.034)
		4	2.225	(3.144)	**2.180**	(3.821)	**0.277**(0.063)	0.289 (0.060)
		5	3.589	(4.506)	**2.107**	(1.770)	**0.298**(0.078)	0.311 (0.062)
		6	14.437	(59.738)	**3.465**	(4.619)	**0.327**(0.078)	0.343 (0.066)
	1000	2	**1.039**	(3.428)	1.655	(7.867)	0.297 (0.016)	**0.296**(0.016)
		3	**0.944**	(2.446)	1.031	(2.911)	0.268 (0.019)	**0.267**(0.020)
		4	1.014	(0.717)	**0.862**	(0.866)	**0.223**(0.029)	0.224 (0.035)
		5	6.850	(29.582)	**1.730**	(2.075)	0.304 (0.525)	**0.267**(0.225)
		6	6.194	(17.516)	**1.552**	(1.226)	0.345 (0.609)	**0.283**(0.223)
	2000	2	**0.407**	(0.452)	0.603	(1.477)	0.299 (0.017)	**0.298**(0.015)
		3	**0.411**	(0.417)	0.446	(0.582)	0.270 (0.020)	**0.267**(0.019)
		4	1.271	(4.686)	**0.630**	(1.170)	0.221 (0.029)	**0.207**(0.030)
		5	3.318	(7.053)	**1.129**	(1.989)	0.260 (0.266)	**0.210**(0.074)
		6	3.021	(4.593)	**1.025**	(0.992)	0.299 (0.317)	**0.220**(0.087)

Note: This table presents the spectral norm errors of the inverse sparse integrated volatility matrix estimators, where the sparse integrated volatility matrix estimators are calculated from the POET procedures with the RPRVM, CRPRVM, PRPRVM, and PCRPRVM estimators under the exponentially decaying correlation with $\rho = 0.5$, varying the number of common factors specified in the POET procedures to 2, 3, 4, 5, and 6 for sensitivity analysis. The true number of common factors is 4.

B.1.2 Sensitivity analysis of sparsity thresholding ω_n

1) $\|\widehat{\Gamma} - \Gamma\|_\Gamma$ calculated under the exponentially decaying correlation and different thresholding levels

Table B.6 Relative Frobenius norm errors under the exponentially decaying correlation and different thresholding levels

		Original			PSD					
p	n	RPRVM	PRVM	CRPRVM	PRPRVM	PPRVM	PCRPRVM1	PCRPRVM2	PCRPRVM3	PCRPRVM4
52	200	1.026(0.131)	1.568(0.077)	**0.957**(0.083)	0.976(0.108)	1.494(0.076)	0.954 (0.080)	0.944 (0.080)	0.937 (0.081)	**0.934** (0.081)
	500	0.793(0.185)	1.302(0.067)	**0.714**(0.064)	0.760(0.134)	1.234(0.055)	0.710 (0.051)	0.706 (0.052)	0.704 (0.052)	**0.704** (0.052)
	1000	0.701(0.237)	1.141(0.092)	**0.616**(0.095)	0.672(0.155)	1.085(0.072)	0.608 (0.086)	0.607 (0.086)	0.607 (0.085)	**0.606** (0.085)
	2000	0.648(0.232)	0.997(0.077)	**0.548**(0.101)	0.638(0.170)	0.956(0.062)	0.542 (0.084)	0.542 (0.084)	0.542 (0.084)	**0.541** (0.084)
100	200	1.300(0.331)	2.271(0.132)	**1.241**(0.221)	1.250(0.285)	2.168(0.114)	1.234 (0.195)	1.217 (0.195)	1.206 (0.193)	**1.202** (0.192)
	500	0.948(0.339)	1.857(0.257)	**0.888**(0.218)	0.887(0.081)	1.717(0.075)	0.847 (0.060)	0.842 (0.060)	0.841 (0.060)	**0.841** (0.060)
	1000	0.708(0.113)	1.555(0.076)	**0.683**(0.085)	0.737(0.114)	1.477(0.073)	0.689 (0.090)	**0.688** (0.089)	0.689 (0.089)	0.690 (0.088)
	2000	0.669(0.269)	1.338(0.064)	**0.585**(0.083)	0.651(0.192)	1.263(0.048)	**0.578** (0.081)	0.579 (0.077)	0.579 (0.073)	0.579 (0.070)
200	200	1.528(0.153)	3.099(0.097)	**1.505**(0.138)	1.480(0.139)	3.011(0.086)	1.509 (0.114)	1.486 (0.114)	1.473 (0.114)	**1.471** (0.115)
	500	1.074(0.171)	2.572(0.132)	**1.049**(0.133)	1.098(0.122)	2.467(0.114)	1.062 (0.104)	1.057 (0.102)	**1.056** (0.101)	1.057 (0.100)
	1000	0.979(0.771)	2.244(0.141)	**0.916**(0.592)	1.008(0.718)	2.135(0.120)	0.929 (0.594)	**0.929** (0.592)	0.930 (0.591)	0.931 (0.591)
	2000	0.710(0.156)	1.902(0.070)	**0.677**(0.091)	0.770(0.092)	1.792(0.059)	**0.690** (0.059)	0.693 (0.059)	0.694 (0.059)	0.695 (0.060)

Note: This table presents the relative Frobenius norm errors of RPRVM-POET, PRVM, CRPRVM-POET, PRPRVM-POET, PPRVM, and PCRPRVM-POET estimators for Γ under the exponentially decaying correlation with $\rho = 0.5$. Columns "PCRPRVM1", "PCRPRVM2", "PCRPRVM3", and "PCRPRVM4" report the performance of the PCRPRVM-POET estimator with thresholding values ω_n corresponding to sparsity levels of approximately 85%, 90%, 95%, and 100%, respectively, where the sparsity level is defined as the number of non-diagonal zero elements of $\widehat{\Sigma}$ divided by the total number of non-diagonal elements.

2) Portfolio risks

Table B.7 Average relative errors under the exponentially decaying correlation and different thresholding levels

		Original			PSD					
p	n	RPRVM	PRVM	CRPRVM	PRPRVM	PPRVM	PCRPRVM1	PCRPRVM2	PCRPRVM3	PCRPRVM4
52	200	0.322(0.137)	0.338(0.144)	**0.281**(0.103)	0.330(0.153)	0.296(0.126)	0.266 (0.100)	0.267 (0.100)	0.268 (0.098)	**0.266** (0.097)
	500	0.260(0.130)	0.260(0.102)	**0.220**(0.065)	0.266(0.146)	0.227(0.102)	0.209 (0.072)	0.208 (0.067)	**0.205** (0.068)	0.210 (0.066)
	1000	0.238(0.130)	0.216(0.084)	**0.185**(0.054)	0.243(0.129)	0.184(0.053)	0.175 (0.048)	**0.175** (0.045)	0.177 (0.049)	0.178 (0.046)
	2000	0.263(0.206)	0.232(0.096)	**0.178**(0.058)	0.272(0.217)	0.216(0.092)	0.172 (0.057)	0.171 (0.057)	**0.171** (0.056)	0.175 (0.055)
100	200	0.305(0.150)	0.354(0.126)	**0.284**(0.079)	0.307(0.182)	0.299(0.137)	0.261 (0.094)	**0.254** (0.085)	0.257 (0.082)	0.257 (0.080)
	500	**0.214**(0.086)	0.284(0.127)	0.218(0.085)	0.219(0.101)	0.239(0.101)	**0.196** (0.079)	0.202 (0.076)	0.200 (0.075)	0.199 (0.079)
	1000	0.208(0.080)	0.245(0.095)	**0.193**(0.053)	0.207(0.096)	0.233(0.095)	**0.181** (0.055)	0.183 (0.056)	0.184 (0.054)	0.184 (0.055)
	2000	0.196(0.094)	0.220(0.104)	**0.174**(0.053)	0.192(0.101)	0.196(0.085)	0.160 (0.048)	0.160 (0.048)	**0.158** (0.047)	0.164 (0.049)
200	200	**0.274**(0.084)	0.369(0.141)	0.279(0.079)	0.266(0.102)	0.333(0.159)	0.253 (0.083)	0.251 (0.085)	**0.248** (0.074)	0.250 (0.078)
	500	0.219(0.068)	0.268(0.096)	**0.219**(0.057)	0.198(0.069)	0.289(0.131)	0.190 (0.053)	0.191 (0.049)	0.189 (0.050)	**0.188** (0.050)
	1000	0.203(0.142)	0.255(0.095)	**0.199**(0.080)	0.204(0.173)	0.289(0.145)	0.181 (0.094)	**0.175** (0.077)	0.176 (0.094)	0.177 (0.088)
	2000	**0.159**(0.059)	0.223(0.110)	0.171(0.060)	0.145(0.062)	0.208(0.104)	0.146 (0.055)	0.145 (0.050)	0.148 (0.056)	**0.144** (0.049)

Note: This table presents the average relative errors of the portfolio risks calculated by the RPRVM-POET, PRVM, CRPRVM-POET, PRPRVM-POET, PPRVM, and PCRPRVM-POET estimators under the exponentially decaying correlation with $\rho = 0.5$. Columns "PCRPRVM1", "PCRPRVM2", "PCRPRVM3", and "PCRPRVM4" report the performance of the PCRPRVM-POET estimator with thresholding values ω_n corresponding to sparsity levels of approximately 85%, 90%, 95%, and 100%, respectively, where the sparsity level is defined as the number of non-diagonal zero elements of $\widehat{\Sigma}$ divided by the total number of non-diagonal elements.

3) $\|\widehat{\Gamma}^{-1} - \Gamma^{-1}\|_2$ calculated under the exponentially decaying correlation and different thresholding levels

Table B.8 Spectral norm errors of the inverse integrated volatility matrix estimators under the exponentially decaying correlation and different thresholding levels

		Original		PSD				
p	n	RPRVM	CRPRVM	PRPRVM	PCRPRVM1	PCRPRVM2	PCRPRVM3	PCRPRVM4
52	200	1.994(1.128)	**1.579**(1.077)	0.495 (0.114)	0.607 (0.157)	0.537 (0.157)	0.498 (0.151)	**0.484**(0.146)
	500	0.910(0.739)	**0.682**(0.531)	0.342 (0.092)	0.349 (0.070)	0.334 (0.071)	0.328 (0.071)	**0.327**(0.072)
	1000	0.556(0.353)	**0.467**(0.460)	0.284 (0.061)	0.286 (0.087)	0.280 (0.081)	0.278 (0.079)	**0.278**(0.078)
	2000	0.451(0.262)	**0.383**(0.561)	0.227 (0.036)	0.221 (0.027)	0.219 (0.028)	0.219 (0.028)	**0.218**(0.028)
100	200	3.279(2.155)	**2.475**(3.500)	**0.397**(0.079)	0.553 (0.075)	0.464 (0.071)	0.422 (0.068)	0.411 (0.067)
	500	1.452(1.162)	**0.859**(0.478)	**0.278**(0.053)	0.319 (0.065)	0.305 (0.064)	0.301 (0.064)	0.300 (0.065)
	1000	0.675(0.620)	**0.651**(0.679)	**0.226**(0.038)	0.241 (0.049)	0.238 (0.047)	0.237 (0.047)	0.237 (0.046)
	2000	0.743(1.675)	**0.466**(0.375)	**0.199**(0.037)	0.208 (0.043)	0.207 (0.043)	0.208 (0.043)	0.209 (0.043)
200	200	5.448(4.802)	**4.695**(5.153)	**0.409**(0.083)	0.611 (0.082)	0.474 (0.079)	0.432 (0.080)	0.422 (0.080)
	500	2.059(3.531)	**1.258**(0.827)	**0.242**(0.035)	0.279 (0.044)	0.263 (0.044)	0.259 (0.044)	0.259 (0.045)
	1000	1.137(2.607)	**0.689**(0.584)	0.216 (0.025)	0.213 (0.037)	**0.211**(0.029)	0.212 (0.029)	0.212 (0.030)
	2000	0.950(1.409)	**0.847**(1.184)	0.219 (0.026)	**0.201**(0.024)	0.203 (0.025)	0.205 (0.025)	0.207 (0.028)

Note: This table presents the spectral norm errors of the inverse integrated volatility matrix estimators calculated using the RPRVM-POET, CRPRVM-POET, PRPRVM-POET, and PCRPRVM-POET estimators under the exponentially decaying correlation with $\rho = 0.5$. Columns "PCRPRVM1", "PCRPRVM2", "PCRPRVM3", and "PCRPRVM4" report the performance of the PCRPRVM-POET estimator with thresholding values ω_n corresponding to sparsity levels of approximately 85%, 90%, 95%, and 100%, respectively, where the sparsity level is defined as the number of non-diagonal zero elements of $\widehat{\Sigma}$ divided by the total number of non-diagonal elements.

4) $\|\widehat{\Sigma} - \Sigma\|_2$ calculated under the exponentially decaying correlation and different thresholding levels

Table B.9 Spectral norm errors of the sparse integrated volatility matrix estimators under the exponentially decaying correlation and different thresholding levels

		Original		PSD				
p	n	RPRVM	CRPRVM	PRPRVM	PCRPRVM1	PCRPRVM2	PCRPRVM3	PCRPRVM4
52	200	8.029 (3.394)	**6.498** (3.360)	4.125 (0.465)	4.309 (0.386)	4.159 (0.364)	4.073 (0.359)	**4.037**(0.344)
	500	6.203 (4.618)	**4.835** (2.684)	3.881 (1.460)	3.900 (1.606)	3.812 (1.559)	3.758 (1.528)	**3.744**(1.516)
	1000	6.052 (5.811)	**4.792** (3.447)	3.930 (1.630)	3.957 (2.319)	3.920 (2.289)	3.904 (2.270)	**3.902**(2.269)
	2000	5.799 (5.194)	**4.594** (3.814)	4.569 (3.791)	4.285 (3.440)	4.220 (3.400)	4.177 (3.375)	**4.134**(3.357)
100	200	14.027 (9.778)	**12.514** (9.190)	7.690 (6.137)	8.646 (6.638)	7.847 (6.450)	7.307 (6.187)	**7.038**(5.945)
	500	11.456(17.553)	**9.088** (12.427)	5.954 (2.930)	5.083 (2.180)	4.874 (2.075)	4.754 (1.989)	**4.714**(1.962)
	1000	6.070 (4.918)	**5.818** (4.614)	5.777 (4.185)	4.788 (4.318)	4.697 (4.249)	**4.651**(4.214)	4.656 (4.213)
	2000	8.040 (7.816)	**5.934** (4.576)	5.171 (3.995)	4.286 (4.373)	4.213 (4.101)	4.157 (3.780)	**4.076**(3.301)
200	200	20.319 (9.882)	**18.193**(11.816)	9.038 (6.446)	10.002(7.707)	8.994 (7.549)	8.357 (7.311)	**8.085**(7.034)
	500	14.276(13.178)	**12.540**(10.792)	11.207(9.746)	9.729 (9.967)	9.441 (9.754)	9.289 (9.548)	**9.190**(9.392)
	1000	14.736(14.304)	**11.565**(10.186)	11.464(9.097)	9.685 (9.704)	9.328 (9.040)	9.026 (8.467)	**8.794**(8.144)
	2000	11.213(13.041)	**9.474** (9.460)	10.415(8.517)	7.391 (6.957)	7.290 (6.845)	7.223 (6.743)	**7.180**(6.661)

Note: This table presents the spectral norm errors of the sparse integrated volatility matrix estimators calculated from the POET procedures with the RPRVM, CRPRVM, PRPRVM, and PCRPRVM estimators under the exponentially decaying correlation with $\rho = 0.5$. Columns "PCRPRVM1", "PCRPRVM2", "PCRPRVM3", and "PCRPRVM4" report the performance of the PCRPRVM-POET estimator with thresholding values ω_n corresponding to sparsity levels of approximately 85%, 90%, 95%, and 100%, respectively, where the sparsity level is defined as the number of non-diagonal zero elements of $\widehat{\Sigma}$ divided by the total number of non-diagonal elements.

5) $\|\widehat{\Sigma}^{-1} - \Sigma^{-1}\|_2$ calculated under the exponentially decaying correlation and different thresholding levels

Table B.10 Spectral norm errors of the inverse sparse integrated volatility matrix estimators under the exponentially decaying correlation and different thresholding levels

		Original		PSD				
p	n	RPRVM	CRPRVM	PRPRVM	PCRPRVM1	PCRPRVM2	PCRPRVM3	PCRPRVM4
52	200	5.601 (11.707)	**3.443** (7.321)	0.588 (0.195)	0.702 (0.208)	0.617 (0.209)	0.569 (0.205)	**0.551**(0.200)
	500	1.122 (0.978)	**0.915** (1.020)	0.393 (0.156)	0.396 (0.132)	0.378 (0.130)	0.370 (0.128)	**0.369**(0.127)
	1000	**0.712** (0.624)	4.276 (27.003)	0.328 (0.088)	0.323 (0.153)	0.315 (0.144)	0.312 (0.140)	**0.311**(0.140)
	2000	1.439 (6.203)	**0.448** (0.830)	0.253 (0.055)	0.237 (0.034)	0.234 (0.036)	0.234 (0.036)	**0.234**(0.036)
100	200	8.029 (11.674)	**5.346**(10.949)	**0.426**(0.098)	0.587 (0.093)	0.491 (0.085)	0.448 (0.083)	0.437 (0.080)
	500	3.866 (10.656)	**1.035** (0.698)	**0.292**(0.061)	0.336 (0.073)	0.320 (0.074)	0.315 (0.074)	0.314 (0.074)
	1000	**0.894** (1.240)	1.087 (2.412)	**0.240**(0.050)	0.255 (0.064)	0.251 (0.063)	0.250 (0.063)	0.250 (0.063)
	2000	2.590 (13.871)	**0.535** (0.555)	**0.210**(0.050)	0.217 (0.048)	0.217 (0.048)	0.217 (0.048)	0.219 (0.049)
200	200	**10.891**(15.294)	19.012(65.074)	**0.425**(0.089)	0.627 (0.088)	0.486 (0.084)	0.444 (0.083)	0.435 (0.083)
	500	3.378 (7.440)	**1.440** (1.079)	**0.247**(0.037)	0.288 (0.048)	0.271 (0.049)	0.266 (0.049)	0.265 (0.049)
	1000	1.341 (3.417)	**0.774** (0.938)	0.223 (0.043)	0.241 (0.190)	0.223 (0.069)	**0.216**(0.033)	0.217 (0.034)
	2000	1.163 (1.921)	**0.954** (1.445)	0.220 (0.027)	**0.204**(0.026)	0.206 (0.026)	0.207 (0.026)	0.210 (0.029)

Note: This table presents the spectral norm errors of the inverse sparse integrated volatility matrix estimators, where the sparse integrated volatility matrix estimators are calculated from the POET procedures with the RPRVM, CRPRVM, PRPRVM, and PCRPRVM estimators under the exponentially decaying correlation with $\rho = 0.5$. Columns "PCRPRVM1", "PCRPRVM2", "PCRPRVM3", and "PCRPRVM4" report the performance of the PCRPRVM-POET estimator with thresholding values ω_n corresponding to sparsity levels of approximately 85%, 90%, 95%, and 100%, respectively, where the sparsity level is defined as the number of non-diagonal zero elements of $\widehat{\Sigma}$ divided by the total number of non-diagonal elements.

B.2 Sensitivity analysis performed in the empirical study

Table B.11 Ex-post variations ($\times 10^4$) of minimum variance portfolios

Variations \ Method / Rank r	RPRVM	PRVM	CRPRVM	RPRVM-POET	CRPRVM-POET1	CRPRVM-POET2	CRPRVM-POET3
2	6.0735	6.7474	**5.6314**	5.9110	5.7512	**5.7438**	5.7611
3	6.0735	6.7474	5.6314	5.9384	5.7575	5.7287	**5.7287**
4	6.0735	6.7474	5.6314	5.5656	5.6249	**5.2026**	5.2956
5	6.0735	6.7474	5.6314	5.5300	5.6011	**5.2193**	5.3421
6	6.0735	6.7474	5.6314	5.5398	5.6116	5.6209	**5.3282**

Note: This table presents the ex-post variations ($\times 10^4$) of minimum variance portfolios constructed by the RPRVM, PRVM, CRPRVM, RPRVM-POET, and CRPRVM-POET estimators with the number of factors $r = 2, 3, 4, 5, 6$. Columns "CRPRVM-POET1", "CRPRVM-POET2", and "CRPRVM-POET3" report the performance of the CRPRVM-POET estimator with thresholding values ω_n corresponding to sparsity levels of approximately 85%, 90%, and 95%, respectively, where the sparsity level is defined as the number of non-diagonal zero elements of $\widehat{\Sigma}$ divided by the total number of non-diagonal elements. The performance of the CRPRVM-POET estimator with thresholding values ω_n corresponding to sparsity levels of approximately 100% is shown in the main text.

Table B.12 Sharpe ratios

Ratios \ Method / Rank r	RPRVM	PRVM	CRPRVM	RPRVM-POET	CRPRVM-POET1	CRPRVM-POET2	CRPRVM-POET3
2	0.0031	-0.0292	**0.0172**	0.0156	0.0114	0.0127	**0.0178**
3	0.0031	-0.0292	0.0172	0.0100	**0.0135**	0.0131	0.0131
4	0.0031	-0.0292	0.0172	0.0117	0.0203	**0.0204**	0.0122
5	0.0031	-0.0292	0.0172	0.0072	0.0158	**0.0174**	0.0088
6	0.0031	-0.0292	0.0172	0.0122	0.0174	**0.0185**	0.0125

Note: This table presents the Sharpe ratios of minimum variance portfolios constructed by the RPRVM, PRVM, CRPRVM, RPRVM-POET, and CRPRVM-POET estimators with the number of factors $r = 2, 3, 4, 5, 6$. Columns "CRPRVM-POET1", "CRPRVM-POET2", and "CRPRVM-POET3" report the performance of the CRPRVM-POET estimator with thresholding values ω_n corresponding to sparsity levels of approximately 85%, 90%, and 95%, respectively, where the sparsity level is defined as the number of non-diagonal zero elements of $\widehat{\Sigma}$ divided by the total number of non-diagonal elements. The performance of the CRPRVM-POET estimator with thresholding values ω_n corresponding to sparsity levels of approximately 100% is shown in the main text.

C Additional results corresponding to the t-distribution

In this section, we report additional results corresponding to when the noise is generated from the t-distribution, including the results of $\|\widehat{\Gamma} - \Gamma\|_\Gamma$ and the average relative errors of the portfolio risks calculated when the microstructure noises have constant correlation and $\{\|\widehat{\Gamma}^{-1} -$

$\Gamma^{-1}\|_2, \|\widehat{\Sigma} - \Sigma\|_2, \|\widehat{\Sigma}^{-1} - \Sigma^{-1}\|_2\}$ calculated under two kinds of cross-sectional dependencies of the microstructure noises.

1) $\|\widehat{\Gamma} - \Gamma\|_\Gamma$ calculated under the constant correlation

Table C.1 Relative Frobenius norm errors under constant correlation with $\rho \in \{0, 0.5, 0.9\}$

		Original			PSD		
p	n	RPRVM	PRVM	CRPRVM	PRPRVM	PPRVM	PCRPRVM
Constant correlation: $\rho = 0$							
52	200	**1.002** (0.119)	1.688 (0.114)	1.005 (0.120)	**0.976** (0.088)	1.585 (0.089)	0.988 (0.088)
	500	**0.755** (0.061)	1.378 (0.077)	0.757 (0.063)	**0.743** (0.049)	1.298 (0.063)	0.749 (0.050)
	1000	**0.616** (0.047)	1.188 (0.050)	0.618 (0.048)	**0.613** (0.041)	1.132 (0.044)	0.615 (0.042)
	2000	**0.548** (0.096)	1.044 (0.083)	0.554 (0.106)	**0.540** (0.071)	0.999 (0.057)	0.541 (0.071)
100	200	**1.216** (0.154)	2.332 (0.143)	1.218 (0.158)	**1.196** (0.162)	2.209 (0.129)	1.217 (0.168)
	500	**0.884** (0.145)	1.944 (0.114)	0.889 (0.146)	**0.900** (0.155)	1.819 (0.106)	0.909 (0.157)
	1000	**0.714** (0.076)	1.662 (0.064)	0.719 (0.080)	**0.718** (0.074)	1.557 (0.057)	0.722 (0.074)
	2000	0.677 (0.389)	1.491 (0.206)	**0.677** (0.388)	**0.676** (0.370)	1.368 (0.077)	0.678 (0.372)
200	200	**1.581** (0.163)	3.328 (0.164)	1.586 (0.168)	**1.525** (0.115)	3.185 (0.115)	1.552 (0.116)
	500	**1.084** (0.085)	2.746 (0.103)	1.102 (0.114)	**1.108** (0.078)	2.605 (0.086)	1.120 (0.078)
	1000	**0.868** (0.135)	2.365 (0.079)	0.873 (0.138)	**0.899** (0.101)	2.226 (0.058)	0.906 (0.101)
	2000	**0.707** (0.099)	2.057 (0.092)	0.710 (0.105)	**0.741** (0.065)	1.912 (0.064)	0.746 (0.066)
Constant correlation: $\rho = 0.5$							
52	200	1.242 (0.329)	1.622 (0.119)	**0.994** (0.166)	1.194 (0.330)	1.536 (0.129)	**0.977** (0.175)
	500	1.179 (0.485)	1.349 (0.102)	**0.744** (0.085)	1.123 (0.424)	1.271 (0.095)	**0.732** (0.090)
	1000	1.189 (0.683)	1.167 (0.085)	**0.667** (0.271)	1.129 (0.639)	1.104 (0.071)	**0.647** (0.271)
	2000	1.067 (0.506)	1.001 (0.073)	**0.536** (0.112)	1.000 (0.435)	0.953 (0.066)	**0.522** (0.098)
100	200	1.625 (0.462)	2.238 (0.118)	**1.192** (0.153)	1.532 (0.432)	2.112 (0.107)	**1.150** (0.121)
	500	1.421 (0.646)	1.872 (0.121)	**0.897** (0.200)	1.338 (0.552)	1.750 (0.106)	**0.868** (0.141)
	1000	1.338 (0.787)	1.614 (0.089)	**0.718** (0.163)	1.280 (0.733)	1.520 (0.088)	**0.726** (0.164)
	2000	1.656 (1.942)	1.867 (3.167)	**0.700** (0.592)	1.330 (0.797)	1.774 (3.180)	**0.606** (0.155)
200	200	2.420 (1.410)	3.226 (0.239)	**1.678** (0.861)	2.266 (1.375)	3.096 (0.215)	**1.639** (0.871)
	500	2.060 (0.930)	2.657 (0.125)	**1.087** (0.146)	1.926 (0.932)	2.510 (0.141)	**1.083** (0.154)
	1000	1.826 (0.887)	2.324 (0.139)	**0.899** (0.202)	1.622 (0.716)	2.163 (0.108)	**0.866** (0.078)
	2000	1.918 (1.903)	2.016 (0.168)	**0.862** (1.019)	1.775 (1.840)	1.858 (0.119)	**0.866** (1.024)
Constant correlation: $\rho = 0.9$							
52	200	1.894 (0.569)	1.552 (0.151)	**0.987** (0.187)	1.781 (0.568)	1.468 (0.175)	**0.950** (0.195)
	500	2.020 (0.612)	1.290 (0.116)	**0.726** (0.114)	1.935 (0.614)	1.215 (0.119)	**0.712** (0.112)
	1000	2.241 (1.242)	1.152 (0.143)	**0.698** (0.516)	2.137 (1.208)	1.073 (0.153)	**0.669** (0.518)
	2000	2.309 (0.713)	0.979 (0.084)	**0.529** (0.076)	2.174 (0.615)	0.898 (0.070)	**0.511** (0.053)
100	200	2.563 (0.854)	2.206 (0.250)	**1.273** (0.375)	2.387 (0.858)	2.088 (0.261)	**1.203** (0.355)
	500	3.273 (1.078)	1.878 (0.178)	**0.931** (0.199)	3.076 (1.004)	1.761 (0.196)	**0.893** (0.192)
	1000	2.993 (0.766)	1.589 (0.115)	**0.711** (0.120)	2.858 (0.748)	1.477 (0.156)	**0.680** (0.109)
	2000	3.351 (1.005)	1.392 (0.144)	**0.617** (0.125)	3.156 (0.986)	1.260 (0.147)	**0.578** (0.092)
200	200	3.844 (1.189)	3.104 (0.191)	**1.619** (0.387)	3.608 (1.258)	2.939 (0.213)	**1.519** (0.373)
	500	4.432 (1.138)	2.614 (0.227)	**1.125** (0.180)	4.219 (1.174)	2.434 (0.272)	**1.040** (0.139)
	1000	4.512 (2.983)	2.238 (0.283)	**0.964** (0.911)	4.206 (2.384)	2.040 (0.148)	**0.866** (0.576)
	2000	4.677 (1.038)	1.914 (0.143)	**0.684** (0.181)	4.478 (1.023)	1.727 (0.108)	**0.636** (0.066)

Note: This table presents the relative Frobenius norm errors of RPRVM-POET, PRVM, CRPRVM-POET, PRPRVM-POET, PPRVM, and PCRPRVM-POET estimators for Γ under constant correlation with $\rho \in \{0, 0.5, 0.9\}$.

2) Portfolio risks

Table C.2 Average relative errors under constant correlation with $\rho \in \{0, 0.5, 0.9\}$

		Original			PSD		
p	n	RPRVM	PRVM	CRPRVM	PRPRVM	PPRVM	PCRPRVM
Constant correlation: $\rho = 0$							
52	200	0.262 (0.067)	**0.250** (0.072)	0.258 (0.067)	0.239 (0.064)	**0.238** (0.077)	0.239 (0.064)
	500	0.199 (0.044)	**0.191** (0.039)	0.198 (0.044)	0.186 (0.042)	**0.180** (0.041)	0.185 (0.042)
	1000	0.173 (0.044)	**0.154** (0.032)	0.173 (0.044)	0.164 (0.040)	**0.149** (0.029)	0.163 (0.040)
	2000	0.141 (0.038)	**0.137** (0.038)	0.141 (0.038)	0.137 (0.036)	**0.134** (0.036)	0.137 (0.036)
100	200	0.251 (0.071)	**0.242** (0.068)	0.249 (0.070)	0.215 (0.072)	0.227 (0.069)	**0.215** (0.071)
	500	0.214 (0.051)	**0.207** (0.054)	0.213 (0.051)	0.184 (0.047)	0.191 (0.055)	**0.184** (0.047)
	1000	0.181 (0.044)	**0.177** (0.041)	0.180 (0.044)	0.160 (0.038)	0.167 (0.043)	**0.159** (0.039)
	2000	0.159 (0.048)	**0.148** (0.044)	0.159 (0.048)	0.142 (0.042)	0.141 (0.044)	**0.141** (0.042)
200	200	0.262 (0.074)	**0.245** (0.054)	0.260 (0.075)	**0.211** (0.049)	0.222 (0.050)	0.213 (0.051)
	500	0.191 (0.046)	**0.190** (0.043)	0.190 (0.046)	0.159 (0.042)	0.181 (0.055)	**0.159** (0.041)
	1000	0.171 (0.047)	**0.169** (0.043)	0.170 (0.047)	**0.143** (0.038)	0.166 (0.048)	0.143 (0.039)
	2000	0.154 (0.044)	**0.137** (0.044)	0.153 (0.044)	0.123 (0.043)	0.140 (0.052)	**0.123** (0.043)
Constant correlation: $\rho = 0.5$							
52	200	0.972 (0.674)	0.534 (0.302)	**0.348** (0.181)	1.005 (0.690)	0.373 (0.225)	**0.326** (0.186)
	500	1.347 (0.899)	0.397 (0.197)	**0.291** (0.121)	1.359 (0.896)	0.314 (0.180)	**0.284** (0.127)
	1000	1.517 (1.113)	0.374 (0.220)	**0.266** (0.258)	1.521 (1.113)	0.321 (0.212)	**0.263** (0.266)
	2000	1.491 (0.933)	0.296 (0.149)	**0.212** (0.108)	1.481 (0.916)	0.270 (0.158)	**0.208** (0.109)
100	200	1.940 (1.328)	0.757 (0.497)	**0.384** (0.199)	1.977 (1.349)	0.563 (0.442)	**0.378** (0.227)
	500	2.153 (1.540)	0.667 (0.554)	**0.359** (0.262)	2.167 (1.549)	0.533 (0.499)	**0.350** (0.277)
	1000	2.079 (1.549)	0.604 (0.350)	**0.290** (0.186)	2.088 (1.546)	0.522 (0.384)	**0.292** (0.198)
	2000	2.661 (2.109)	0.650 (0.447)	**0.291** (0.233)	2.595 (2.018)	0.540 (0.459)	**0.275** (0.206)
200	200	3.984 (3.573)	1.288 (0.815)	**0.810** (1.339)	3.980 (3.620)	1.286 (0.976)	**0.856** (1.360)
	500	4.259 (3.242)	1.269 (1.011)	**0.503** (0.328)	4.229 (3.278)	1.125 (0.940)	**0.511** (0.364)
	1000	3.851 (3.157)	1.279 (0.969)	**0.462** (0.340)	3.781 (3.143)	1.079 (1.048)	**0.468** (0.361)
	2000	4.074 (4.001)	1.023 (0.778)	**0.441** (0.973)	4.031 (4.003)	0.870 (0.695)	**0.436** (0.983)
Constant correlation: $\rho = 0.9$							
52	200	2.563 (1.199)	0.714 (0.540)	**0.443** (0.384)	2.594 (1.219)	0.488 (0.460)	**0.411** (0.388)
	500	3.309 (1.297)	0.644 (0.471)	**0.387** (0.283)	3.325 (1.310)	0.496 (0.373)	**0.380** (0.287)
	1000	3.766 (2.141)	0.597 (0.428)	**0.424** (0.949)	3.762 (2.142)	0.416 (0.413)	**0.403** (0.952)
	2000	3.825 (1.177)	0.460 (0.294)	**0.257** (0.127)	3.810 (1.174)	0.325 (0.236)	**0.251** (0.130)
100	200	4.802 (2.394)	1.293 (1.066)	**0.765** (0.961)	4.799 (2.452)	0.938 (0.993)	**0.722** (0.974)
	500	6.778 (2.332)	1.187 (0.778)	**0.599** (0.490)	6.770 (2.363)	0.981 (0.800)	**0.595** (0.509)
	1000	6.563 (1.889)	1.014 (0.576)	**0.374** (0.217)	6.557 (1.901)	0.696 (0.582)	**0.359** (0.228)
	2000	7.281 (2.404)	0.868 (0.623)	**0.320** (0.173)	7.248 (2.431)	0.580 (0.564)	**0.314** (0.184)
200	200	10.786 (5.335)	2.279 (1.633)	**1.323** (1.660)	10.718 (5.471)	1.760 (1.486)	**1.305** (1.703)
	500	13.233 (3.971)	2.286 (1.538)	**0.893** (0.657)	13.205 (4.067)	1.673 (1.507)	**0.889** (0.695)
	1000	12.742 (5.289)	1.483 (1.032)	**0.669** (1.139)	12.758 (5.689)	1.049 (0.812)	**0.645** (1.052)
	2000	13.872 (3.966)	1.475 (1.127)	**0.358** (0.198)	13.845 (4.007)	0.818 (0.753)	**0.347** (0.204)

Note: This table presents the average relative errors of the portfolio risks calculated by the RPRVM-POET, PRVM, CRPRVM-POET, PRPRVM-POET, PPRVM, and PCRPRVM-POET estimators under constant correlation with $\rho \in \{0, 0.5, 0.9\}$.

3) $\|\widehat{\Gamma}^{-1} - \Gamma^{-1}\|_2$ calculated under two kinds of cross-sectional dependencies

Table C.3 Spectral norm errors of the inverse integrated volatility matrix estimators under the exponentially decaying correlation with $\rho \in \{0, 0.5, 0.9\}$

		Original		PSD	
p	n	RPRVM	CRPRVM	PRPRVM	PCRPRVM
Exponential decay: $\rho = 0$					
52	200	1.683 (1.443)	**1.270** (0.903)	0.487 (0.110)	**0.443** (0.096)
	500	0.805 (1.228)	**0.568** (0.327)	0.333 (0.073)	**0.315** (0.073)
	1000	0.472 (0.492)	**0.413** (0.207)	0.273 (0.051)	**0.265** (0.051)
	2000	0.437 (0.339)	**0.429** (0.325)	0.245 (0.054)	**0.239** (0.051)
100	200	2.421 (1.313)	**1.612** (0.904)	0.400 (0.076)	**0.378** (0.071)
	500	**1.080** (1.437)	1.090 (1.548)	0.284 (0.065)	**0.269** (0.065)
	1000	**0.609** (0.540)	0.731 (1.071)	0.227 (0.036)	**0.222** (0.033)
	2000	0.537 (0.977)	**0.491** (0.746)	0.200 (0.027)	**0.198** (0.026)
200	200	4.904 (4.710)	**3.859** (3.425)	0.436 (0.133)	**0.421** (0.130)
	500	1.966 (2.698)	**1.515** (1.522)	0.251 (0.042)	**0.246** (0.042)
	1000	0.678 (0.416)	**0.630** (0.358)	0.214 (0.028)	**0.213** (0.025)
	2000	3.013 (11.018)	**1.429** (4.533)	**0.202** (0.019)	0.203 (0.019)
Exponential decay: $\rho = 0.5$					
52	200	3.667 (9.722)	**1.487** (1.028)	0.518 (0.135)	**0.488** (0.126)
	500	1.407 (3.730)	**0.973** (1.734)	**0.319** (0.077)	0.320 (0.073)
	1000	0.641 (0.506)	**0.467** (0.324)	0.271 (0.066)	**0.261** (0.046)
	2000	0.713 (0.704)	**0.356** (0.183)	0.232 (0.048)	**0.225** (0.042)
100	200	3.469 (3.233)	**2.172** (2.038)	**0.399** (0.091)	0.449 (0.112)
	500	1.376 (1.278)	**0.918** (0.652)	**0.265** (0.051)	0.277 (0.053)
	1000	0.844 (0.785)	**0.717** (0.874)	**0.234** (0.044)	0.253 (0.053)
	2000	0.505 (0.412)	**0.429** (0.321)	0.203 (0.030)	**0.199** (0.032)
200	200	4.592 (3.187)	**4.127** (4.368)	**0.402** (0.085)	0.411 (0.086)
	500	1.777 (1.634)	**1.406** (1.298)	**0.249** (0.041)	0.269 (0.050)
	1000	1.168 (1.490)	**1.113** (1.665)	**0.219** (0.026)	0.220 (0.031)
	2000	0.513 (0.331)	**0.466** (0.249)	0.217 (0.026)	**0.201** (0.027)
Exponential decay: $\rho = 0.9$					
52	200	**2.760** (1.986)	2.847 (7.161)	0.651 (0.242)	**0.474** (0.089)
	500	1.837 (1.195)	**0.712** (0.581)	0.501 (0.123)	**0.332** (0.060)
	1000	1.352 (1.172)	**0.418** (0.232)	0.479 (0.189)	**0.266** (0.046)
	2000	1.776 (1.346)	**0.465** (0.814)	0.546 (0.201)	**0.225** (0.043)
100	200	4.143 (2.729)	**2.479** (1.850)	**0.445** (0.084)	0.486 (0.121)
	500	1.896 (1.422)	**0.873** (0.610)	0.339 (0.100)	**0.304** (0.074)
	1000	1.578 (1.036)	**0.552** (0.517)	0.270 (0.061)	**0.231** (0.032)
	2000	1.362 (1.252)	**0.393** (0.318)	0.270 (0.077)	**0.194** (0.024)
200	200	6.407 (5.172)	**3.652** (3.525)	**0.397** (0.078)	0.464 (0.117)
	500	2.433 (2.085)	**1.624** (1.448)	**0.252** (0.040)	0.295 (0.040)
	1000	1.438 (1.435)	**0.868** (0.705)	**0.222** (0.029)	0.237 (0.033)
	2000	1.269 (1.362)	**0.867** (1.080)	0.228 (0.027)	**0.198** (0.027)

Note: This table presents the spectral norm errors of the inverse integrated volatility matrix estimators calculated using the RPRVM-POET, CRPRVM-POET, PRPRVM-POET, and PCRPRVM-POET estimators under the exponentially decaying correlation with $\rho \in \{0, 0.5, 0.9\}$.

Table C.4 Spectral norm errors of the inverse integrated volatility matrix estimators under constant correlation with $\rho \in \{0, 0.5, 0.9\}$

		Original		PSD	
p	n	RPRVM	CRPRVM	PRPRVM	PCRPRVM
Constant correlation: $\rho = 0$					
52	200	1.819 (1.141)	**1.262** (0.664)	0.499 (0.107)	**0.448** (0.095)
	500	0.739 (0.522)	**0.671** (0.406)	0.365 (0.100)	**0.350** (0.097)
	1000	0.476 (0.331)	**0.459** (0.311)	0.278 (0.052)	**0.270** (0.053)
	2000	0.338 (0.195)	**0.330** (0.192)	0.234 (0.036)	**0.228** (0.035)
100	200	3.245 (2.683)	**2.243** (1.514)	0.428 (0.101)	**0.402** (0.096)
	500	**0.977** (0.782)	1.112 (1.701)	0.287 (0.084)	**0.272** (0.080)
	1000	**0.630** (0.858)	0.646 (1.019)	0.238 (0.049)	**0.230** (0.045)
	2000	0.549 (0.639)	**0.543** (0.616)	0.200 (0.033)	**0.197** (0.032)
200	200	4.861 (4.310)	**3.920** (4.726)	0.421 (0.104)	**0.399** (0.097)
	500	2.438 (3.065)	**1.563** (1.337)	0.254 (0.043)	**0.246** (0.041)
	1000	1.067 (1.999)	**0.915** (1.242)	0.211 (0.019)	**0.210** (0.019)
	2000	0.632 (0.599)	**0.594** (0.517)	0.202 (0.021)	**0.202** (0.021)
Constant correlation: $\rho = 0.5$					
52	200	2.695 (1.990)	**1.417** (0.857)	0.576 (0.162)	**0.481** (0.115)
	500	1.387 (1.094)	**0.625** (0.314)	0.434 (0.107)	**0.320** (0.065)
	1000	1.425 (2.125)	**0.505** (0.599)	0.425 (0.188)	**0.264** (0.049)
	2000	0.900 (0.724)	**0.340** (0.213)	0.379 (0.116)	**0.229** (0.042)
100	200	3.944 (2.246)	**2.272** (1.785)	0.492 (0.138)	**0.456** (0.111)
	500	1.870 (1.391)	**1.058** (1.187)	0.326 (0.074)	**0.274** (0.047)
	1000	1.167 (0.780)	**0.594** (0.716)	0.290 (0.068)	**0.240** (0.044)
	2000	1.164 (0.967)	**0.404** (0.362)	**0.281** (0.106)	0.348 (1.112)
200	200	6.338 (4.161)	**3.961** (5.568)	**0.419** (0.077)	0.441 (0.069)
	500	3.078 (2.742)	**1.411** (1.694)	0.286 (0.062)	**0.280** (0.053)
	1000	2.427 (2.739)	**1.260** (2.380)	0.234 (0.025)	**0.222** (0.033)
	2000	1.776 (2.049)	**0.789** (1.004)	0.234 (0.052)	**0.205** (0.028)
Constant correlation: $\rho = 0.9$					
52	200	3.500 (2.223)	**1.355** (1.212)	0.893 (0.309)	**0.534** (0.127)
	500	2.467 (1.433)	**0.662** (0.742)	0.836 (0.295)	**0.321** (0.087)
	1000	2.823 (4.347)	**0.947** (2.747)	1.085 (0.653)	**0.245** (0.032)
	2000	3.623 (5.117)	**0.427** (0.420)	0.996 (0.491)	**0.216** (0.033)
100	200	6.822 (7.337)	**3.455** (11.066)	0.710 (0.309)	**0.528** (0.126)
	500	4.088 (5.836)	**1.084** (1.664)	0.720 (0.361)	**0.320** (0.064)
	1000	2.699 (1.868)	**0.718** (0.777)	0.879 (0.657)	**0.257** (0.046)
	2000	3.017 (2.368)	**0.509** (0.532)	0.993 (0.538)	**0.205** (0.033)
200	200	7.118 (2.897)	**3.697** (2.835)	0.573 (0.131)	**0.516** (0.104)
	500	5.358 (3.684)	**2.456** (4.168)	0.598 (0.317)	**0.331** (0.049)
	1000	4.117 (2.371)	**1.104** (1.617)	0.629 (0.313)	**0.260** (0.046)
	2000	4.772 (3.152)	**0.779** (1.219)	0.779 (0.393)	**0.207** (0.028)

Note: This table presents the spectral norm errors of the inverse integrated volatility matrix estimators calculated using the RPRVM-POET, CRPRVM-POET, PRPRVM-POET, and PCRPRVM-POET estimators under constant correlation with $\rho \in \{0, 0.5, 0.9\}$.

4) $\|\widehat{\Sigma} - \Sigma\|_2$ calculated under two kinds of cross-sectional dependencies

Table C.5 Spectral norm errors of the sparse integrated volatility matrix estimators under the exponentially decaying correlation with $\rho \in \{0, 0.5, 0.9\}$

		Original		PSD	
p	n	RPRVM	CRPRVM	PRPRVM	PCRPRVM
Exponential decay: $\rho = 0$					
52	200	6.548 (3.060)	**6.101** (2.955)	4.312 (1.254)	**4.250** (1.196)
	500	**5.786** (4.418)	5.903 (4.603)	4.590 (3.930)	**4.546** (3.939)
	1000	**4.406** (2.513)	4.443 (2.638)	3.911 (1.869)	**3.905** (1.876)
	2000	4.966 (4.568)	**4.961** (4.598)	4.078 (3.794)	**4.066** (3.766)
100	200	10.999 (7.106)	**10.521** (7.497)	**6.399** (5.054)	6.449 (5.108)
	500	6.796 (4.944)	**6.595** (5.161)	**4.771** (2.911)	4.814 (2.859)
	1000	7.306 (7.591)	**7.259** (7.682)	**5.994** (6.250)	6.042 (6.266)
	2000	**5.088** (4.589)	5.118 (4.692)	**4.866** (4.615)	4.899 (4.673)
200	200	30.017 (78.382)	**28.603** (77.420)	**10.721** (6.992)	10.842 (6.937)
	500	**12.999** (9.554)	13.105 (9.918)	**8.910** (5.503)	9.078 (5.549)
	1000	**13.272** (14.494)	13.323 (14.549)	**11.645** (14.276)	11.709 (14.240)
	2000	**8.328** (7.360)	8.690 (7.774)	**8.141** (5.738)	8.222 (5.766)
Exponential decay: $\rho = 0.5$					
52	200	9.178 (7.904)	**7.262** (6.049)	**4.445** (1.154)	4.587 (1.413)
	500	7.512 (5.517)	**5.081** (2.634)	3.868 (1.348)	**3.840** (1.480)
	1000	6.605 (6.673)	**5.024** (4.513)	3.759 (1.057)	**3.352** (0.858)
	2000	6.554 (7.811)	**3.953** (1.851)	3.586 (1.191)	**3.349** (1.301)
100	200	13.356 (8.406)	**10.863** (6.813)	5.945 (3.147)	**5.499** (2.686)
	500	10.624 (10.871)	**8.895** (7.970)	6.738 (5.474)	**5.859** (5.276)
	1000	8.470 (9.283)	**6.386** (5.284)	5.505 (2.700)	**4.266** (2.108)
	2000	6.598 (9.351)	**4.742** (4.596)	5.094 (2.477)	**3.292** (0.775)
200	200	21.499 (16.453)	**17.885** (13.572)	11.037 (8.744)	**9.901** (8.071)
	500	12.176 (10.449)	**10.949** (8.779)	10.032 (5.970)	**7.872** (5.171)
	1000	13.361 (16.296)	**10.812** (11.913)	9.665 (6.356)	**6.905** (4.724)
	2000	10.808 (13.654)	**8.132** (6.392)	10.630 (7.153)	**7.315** (5.298)
Exponential decay: $\rho = 0.9$					
52	200	12.510 (7.891)	**8.430** (7.747)	4.432 (0.554)	**4.045** (0.409)
	500	10.472 (5.015)	**6.117** (4.366)	4.528 (1.225)	**3.532** (0.531)
	1000	9.992 (5.075)	**4.058** (1.880)	4.783 (1.406)	**3.340** (0.838)
	2000	12.155 (7.500)	**4.735** (4.108)	4.636 (1.175)	**3.121** (0.466)
100	200	19.773 (7.452)	**10.689** (7.076)	5.050 (1.384)	**4.620** (1.315)
	500	17.370 (12.090)	**8.727** (8.697)	6.019 (3.710)	**4.216** (2.497)
	1000	17.313 (14.798)	**7.249** (7.866)	5.795 (2.249)	**4.488** (5.419)
	2000	19.841 (13.388)	**6.084** (5.851)	6.051 (2.737)	**3.535** (1.987)
200	200	29.030 (14.097)	**21.227** (12.363)	8.340 (3.722)	**7.275** (7.739)
	500	22.394 (21.376)	**12.850** (10.687)	11.533 (6.997)	**6.619** (5.864)
	1000	26.172 (26.917)	**11.110** (9.446)	11.223 (7.720)	**6.134** (6.768)
	2000	25.037 (31.303)	**9.559** (14.344)	12.887 (8.878)	**6.496** (7.365)

Note: This table presents the spectral norm errors of the sparse integrated volatility matrix estimators calculated from the POET procedures with the RPRVM, CRPRVM, PRPRVM, and PCRPRVM estimators under the exponentially decaying correlation with $\rho \in \{0, 0.5, 0.9\}$.

Table C.6 Spectral norm errors of the sparse integrated volatility matrix estimators under constant correlation with $\rho \in \{0, 0.5, 0.9\}$

		Original		PSD	
p	n	RPRVM	CRPRVM	PRPRVM	PCRPRVM
Constant correlation: $\rho = 0$					
52	200	6.893 (4.361)	**6.478** (4.454)	4.258 (0.682)	**4.215** (0.783)
	500	4.892 (2.756)	**4.775** (2.784)	3.777 (0.709)	**3.767** (0.754)
	1000	3.977 (1.463)	**3.953** (1.488)	3.366 (0.770)	**3.347** (0.803)
	2000	**4.183** (2.902)	4.343 (3.201)	3.589 (1.602)	**3.568** (1.587)
100	200	10.438 (6.250)	**9.280** (6.763)	**6.494** (5.542)	6.509 (5.467)
	500	7.940 (6.877)	**7.847** (6.941)	**6.725** (6.432)	6.773 (6.432)
	1000	**6.601** (5.050)	6.785 (5.306)	**5.044** (4.407)	5.076 (4.398)
	2000	7.748 (9.389)	**7.729** (9.285)	6.392 (8.230)	**6.391** (8.211)
200	200	20.596 (12.900)	**19.590** (13.602)	**12.195** (9.778)	12.332 (9.753)
	500	**12.877** (9.123)	13.410 (11.162)	**9.720** (8.255)	9.901 (8.338)
	1000	**10.082** (11.792)	10.197 (11.954)	**8.660** (9.553)	8.770 (9.517)
	2000	**9.252** (7.971)	9.330 (8.167)	**7.715** (5.154)	7.789 (5.158)
Constant correlation: $\rho = 0.5$					
52	200	8.832 (3.994)	**6.335** (3.063)	4.460 (0.920)	**4.211** (0.603)
	500	8.243 (5.376)	**4.966** (3.020)	4.300 (1.516)	**3.611** (0.687)
	1000	8.079 (6.047)	**5.143** (3.636)	4.103 (1.042)	**3.545** (1.057)
	2000	7.586 (5.907)	**4.289** (3.251)	4.072 (1.121)	**3.305** (1.622)
100	200	16.205 (9.586)	**12.373** (8.760)	**5.286** (1.956)	5.560 (3.060)
	500	14.248 (10.259)	**9.141** (8.273)	5.390 (2.447)	**4.642** (3.090)
	1000	11.602 (8.483)	**5.195** (3.174)	5.195 (2.023)	**4.358** (2.532)
	2000	23.057 (63.189)	**9.936** (22.598)	5.854 (2.254)	**4.263** (4.615)
200	200	32.052 (16.027)	**20.218** (13.421)	**8.923** (5.010)	9.046 (6.519)
	500	28.675 (17.142)	**13.837** (13.171)	8.891 (4.419)	**8.527** (11.009)
	1000	29.968 (29.355)	**13.164** (16.784)	9.364 (6.012)	**7.352** (5.942)
	2000	28.198 (22.800)	**11.034** (12.378)	10.490 (9.290)	**8.480** (9.201)
Constant correlation: $\rho = 0.9$					
52	200	12.072 (3.514)	**7.743** (5.549)	4.740 (0.772)	**4.301** (1.285)
	500	10.494 (3.050)	**4.332** (2.901)	4.810 (1.067)	**3.404** (0.328)
	1000	11.348 (4.729)	**5.108** (4.350)	4.962 (1.131)	**3.229** (0.563)
	2000	12.586 (6.243)	**4.179** (3.024)	4.986 (1.200)	**3.085** (0.540)
100	200	22.321 (10.095)	**12.960** (16.106)	5.650 (2.104)	**4.204** (0.756)
	500	24.016 (10.940)	**8.419** (9.585)	6.436 (1.975)	**4.086** (2.866)
	1000	19.731 (7.517)	**6.616** (6.687)	6.652 (2.103)	**3.313** (0.777)
	2000	23.431 (9.215)	**7.259** (7.641)	6.821 (1.875)	**3.577** (2.340)
200	200	41.659 (13.349)	**23.682** (23.454)	6.805 (1.955)	**5.000** (2.060)
	500	42.391 (13.576)	**18.301** (19.543)	8.017 (2.252)	**4.845** (4.139)
	1000	52.230 (64.905)	**14.497** (23.327)	8.282 (2.685)	**3.715** (1.294)
	2000	48.423 (18.153)	**9.845** (15.391)	10.002 (3.184)	**3.800** (2.661)

Note: This table presents the spectral norm errors of the sparse integrated volatility matrix estimators calculated from the POET procedures with the RPRVM, CRPRVM, PRPRVM, and PCRPRVM estimators under constant correlation with $\rho \in \{0, 0.5, 0.9\}$.

5) $\|\widehat{\Sigma}^{-1} - \Sigma^{-1}\|_2$ calculated under two kinds of cross-sectional dependencies

Table C.7 Spectral norm errors of the inverse sparse integrated volatility matrix estimators under the exponentially decaying correlation with $\rho \in \{0, 0.5, 0.9\}$

		Original		PSD	
p	n	RPRVM	CRPRVM	PRPRVM	PCRPRVM
Exponential decay: $\rho = 0$					
52	200	5.546 (18.231)	**2.332** (3.190)	0.541 (0.157)	**0.486** (0.126)
	500	2.114 (8.740)	**0.741** (0.668)	0.380 (0.115)	**0.352** (0.101)
	1000	0.832 (2.723)	**0.508** (0.582)	0.299 (0.061)	**0.289** (0.060)
	2000	0.588 (0.866)	**0.522** (0.503)	0.270 (0.072)	**0.263** (0.068)
100	200	3.949 (3.667)	**2.990** (4.730)	0.431 (0.101)	**0.403** (0.087)
	500	**1.875** (3.727)	4.540 (19.727)	0.309 (0.086)	**0.292** (0.082)
	1000	**0.723** (0.751)	1.136 (2.724)	0.239 (0.041)	**0.233** (0.037)
	2000	0.620 (1.302)	**0.548** (0.921)	0.207 (0.032)	**0.204** (0.031)
200	200	**6.886** (7.306)	9.388 (29.246)	0.452 (0.142)	**0.434** (0.139)
	500	9.441 (51.773)	**2.035** (2.585)	0.260 (0.048)	**0.253** (0.047)
	1000	0.732 (0.495)	**0.675** (0.416)	0.218 (0.031)	**0.216** (0.028)
	2000	4.978 (17.488)	**1.688** (5.362)	0.204 (0.020)	**0.204** (0.020)
Exponential decay: $\rho = 0.5$					
52	200	29.972 (144.604)	**2.980** (5.471)	0.682 (0.569)	**0.589** (0.291)
	500	31.779 (211.499)	**2.680** (6.856)	0.421 (0.309)	**0.356** (0.091)
	1000	0.965 (1.897)	**0.620** (0.818)	0.400 (0.642)	**0.286** (0.060)
	2000	0.980 (1.082)	**0.410** (0.298)	0.267 (0.086)	**0.247** (0.061)
100	200	5.679 (9.374)	**3.251** (4.297)	**0.433** (0.150)	0.487 (0.171)
	500	3.014 (6.918)	**1.441** (2.009)	**0.285** (0.071)	0.291 (0.061)
	1000	1.193 (1.594)	**0.993** (1.931)	**0.248** (0.055)	0.266 (0.062)
	2000	0.558 (0.470)	**0.475** (0.402)	0.211 (0.038)	**0.209** (0.037)
200	200	8.686 (11.906)	**7.333** (12.411)	**0.418** (0.100)	0.424 (0.096)
	500	2.689 (3.909)	**1.748** (2.209)	**0.256** (0.045)	0.277 (0.054)
	1000	1.470 (2.172)	**1.294** (2.156)	**0.223** (0.028)	0.225 (0.034)
	2000	0.550 (0.426)	**0.491** (0.290)	0.219 (0.027)	**0.204** (0.027)
Exponential decay: $\rho = 0.9$					
52	200	7.234 (10.498)	**2.488** (2.660)	0.770 (0.363)	**0.537** (0.140)
	500	3.751 (5.330)	**0.924** (1.062)	0.619 (0.280)	**0.372** (0.099)
	1000	2.099 (2.653)	**0.471** (0.279)	0.574 (0.292)	**0.297** (0.062)
	2000	6.428 (21.692)	**0.598** (1.035)	0.769 (0.598)	**0.247** (0.058)
100	200	12.495 (37.418)	**4.216** (4.729)	**0.494** (0.147)	0.518 (0.138)
	500	3.047 (3.943)	**1.312** (1.908)	0.374 (0.135)	**0.316** (0.078)
	1000	2.544 (2.625)	**0.681** (0.901)	0.300 (0.090)	**0.240** (0.039)
	2000	2.464 (4.276)	**0.426** (0.402)	0.323 (0.205)	**0.202** (0.029)
200	200	12.615 (22.206)	**5.999** (9.091)	**0.414** (0.085)	0.480 (0.131)
	500	3.412 (4.244)	**2.502** (3.779)	**0.261** (0.047)	0.303 (0.044)
	1000	1.844 (2.209)	**0.957** (0.830)	**0.226** (0.030)	0.243 (0.037)
	2000	**1.498** (1.870)	12.688 (83.418)	0.231 (0.030)	**0.201** (0.028)

Note: This table presents the spectral norm errors of the inverse sparse integrated volatility matrix estimators, where the sparse integrated volatility matrix estimators are calculated from the POET procedures with the RPRVM, CRPRVM, PRPRVM, and PCRPRVM estimators under the exponentially decaying correlation with $\rho \in \{0, 0.5, 0.9\}$.

Table C.8 Spectral norm errors of the inverse sparse integrated volatility matrix estimators under constant correlation with $\rho \in \{0, 0.5, 0.9\}$

		Original		PSD	
p	n	RPRVM	CRPRVM	PRPRVM	PCRPRVM
Constant correlation: $\rho = 0$					
52	200	8.977 (37.233)	**2.857** (6.209)	0.579 (0.152)	**0.513** (0.126)
	500	1.261 (1.892)	**1.086** (1.556)	0.402 (0.132)	**0.382** (0.126)
	1000	1.012 (3.731)	**0.744** (1.947)	0.303 (0.068)	**0.295** (0.071)
	2000	0.441 (0.513)	**0.373** (0.243)	0.256 (0.050)	**0.249** (0.048)
100	200	8.072 (17.080)	**6.956** (25.191)	0.460 (0.131)	**0.429** (0.123)
	500	**1.294** (1.376)	3.124 (13.724)	0.306 (0.094)	**0.288** (0.088)
	1000	**0.996** (3.025)	1.839 (8.718)	0.252 (0.062)	**0.243** (0.057)
	2000	0.674 (1.023)	**0.668** (0.982)	0.206 (0.033)	**0.203** (0.033)
200	200	**8.072** (9.902)	10.418 (28.375)	0.439 (0.117)	**0.413** (0.104)
	500	5.222 (9.592)	**2.385** (3.061)	0.264 (0.052)	**0.256** (0.050)
	1000	1.469 (4.356)	**1.057** (1.789)	0.214 (0.021)	**0.213** (0.021)
	2000	0.710 (0.809)	**0.656** (0.668)	**0.204** (0.021)	0.204 (0.022)
Constant correlation: $\rho = 0.5$					
52	200	9.075 (25.679)	**2.999** (6.242)	0.705 (0.273)	**0.549** (0.154)
	500	4.060 (7.477)	**0.777** (0.488)	0.538 (0.305)	**0.360** (0.100)
	1000	2.283 (4.940)	**0.875** (2.486)	0.677 (0.893)	**0.295** (0.076)
	2000	1.754 (3.758)	**0.374** (0.261)	0.437 (0.161)	**0.246** (0.051)
100	200	7.058 (7.071)	**3.965** (5.341)	0.532 (0.181)	**0.481** (0.133)
	500	**3.067** (3.612)	5.492 (29.523)	0.401 (0.385)	**0.289** (0.054)
	1000	1.936 (2.942)	**0.657** (0.827)	0.319 (0.107)	**0.253** (0.053)
	2000	2.097 (3.618)	**0.443** (0.421)	**0.312** (0.151)	0.370 (1.228)
200	200	10.580 (10.905)	**5.387** (9.505)	**0.436** (0.087)	0.455 (0.075)
	500	4.256 (4.380)	**1.897** (2.568)	0.298 (0.072)	**0.290** (0.058)
	1000	3.774 (7.803)	**1.547** (3.404)	0.240 (0.028)	**0.226** (0.035)
	2000	2.901 (4.762)	**1.028** (1.600)	0.250 (0.121)	**0.207** (0.029)
Constant correlation: $\rho = 0.9$					
52	200	6.660 (7.706)	**1.884** (2.431)	1.135 (0.670)	**0.620** (0.212)
	500	5.095 (6.467)	**1.139** (2.549)	1.082 (0.476)	**0.354** (0.132)
	1000	3.718 (4.538)	**1.211** (3.236)	1.776 (1.429)	**0.268** (0.052)
	2000	7.552 (24.691)	**0.905** (3.200)	3.449 (12.766)	**0.236** (0.043)
100	200	16.597 (44.296)	**3.556** (4.310)	0.852 (0.628)	**0.559** (0.139)
	500	5.250 (6.747)	**1.978** (6.760)	1.402 (1.582)	**0.340** (0.076)
	1000	3.573 (2.700)	**0.894** (1.236)	1.508 (1.795)	**0.272** (0.055)
	2000	3.711 (2.635)	**0.610** (1.001)	1.763 (2.318)	**0.213** (0.039)
200	200	10.792 (8.124)	**5.084** (4.441)	0.621 (0.190)	**0.532** (0.111)
	500	**8.018** (7.287)	429.328 (2987.079)	0.739 (0.638)	**0.341** (0.055)
	1000	5.522 (4.284)	**1.394** (3.106)	2.878 (14.655)	**0.265** (0.049)
	2000	6.156 (4.304)	**0.909** (1.763)	1.028 (0.801)	**0.212** (0.030)

Note: This table presents the spectral norm errors of the inverse sparse integrated volatility matrix estimators, where the sparse integrated volatility matrix estimators are calculated from the POET procedures with the RPRVM, CRPRVM, PRPRVM, and PCRPRVM estimators under constant correlation with $\rho \in \{0, 0.5, 0.9\}$.

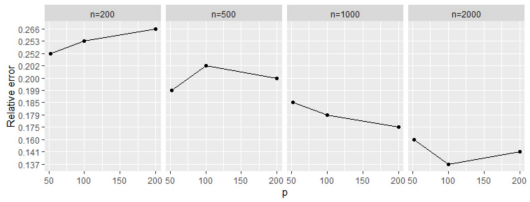
D Additional explanation of Table 3 in the main text

In this section, we explain the reason for the absence of monotonicity of the average relative errors of the portfolio risks against p in Table 3 in the main text. Recall that the relative error of portfolio risk is defined as

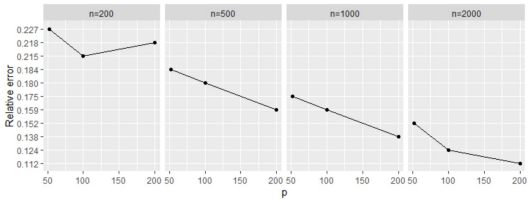
$$\frac{|\omega^{\mathrm{T}}(\widehat{\Gamma} - \Gamma)\omega|}{\omega^{\mathrm{T}}\Gamma\omega}.$$

To give a convincing explanation, we first visualize Table 3 in Section D.1, plotting the average relative errors against the dimensionality p under the exponentially decaying cross-sectional correlation with $\rho \in \{0, 0.5, 0.9\}$. Then, in Section D.2, we plot $|\omega^{\mathrm{T}}(\widehat{\Gamma} - \Gamma)\omega|$ against the dimensionality p under the exponentially decaying cross-sectional correlation with $\rho = 0.5$. The results show that the numerator of the relative error, $|\omega^{\mathrm{T}}(\widehat{\Gamma} - \Gamma)\omega|$, is monotonically decreasing against dimensionality p. That is, it is the denominator of the relative error that causes the lack of monotonicity of the average relative errors against p.

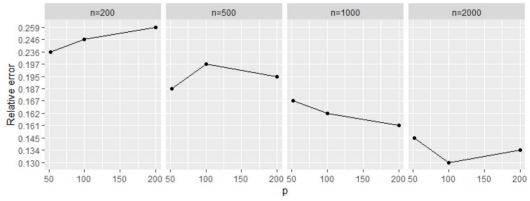
D.1 Visualization of Table 3 in the main text



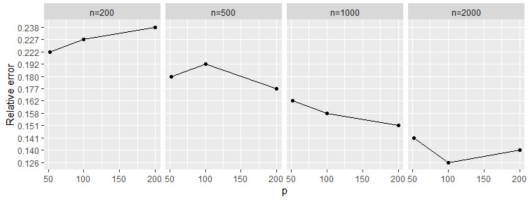
(a) $\rho = 0$, RPRVM-POET



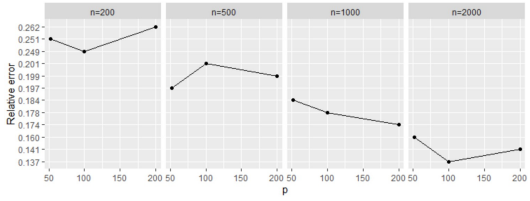
(b) $\rho = 0$, PRPRVM-POET



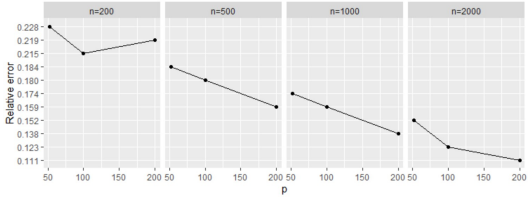
(c) $\rho = 0$, PRVM



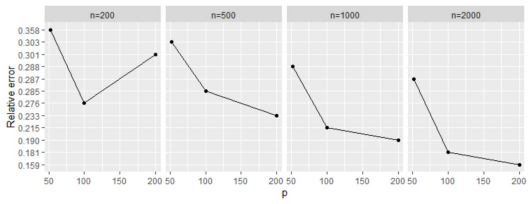
(d) $\rho = 0$, PPRVM



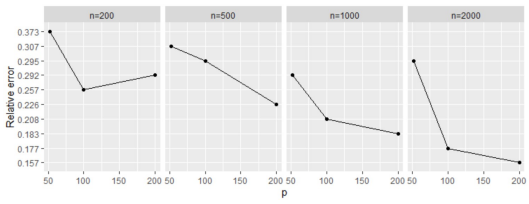
(e) $\rho = 0$, CRPRVM-POET



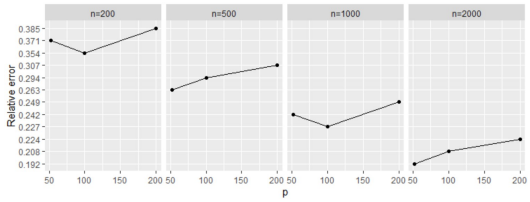
(f) $\rho = 0$, PCRPRVM-POET



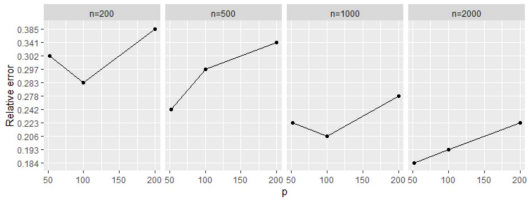
(g) $\rho = 0.5$, RPRVM-POET



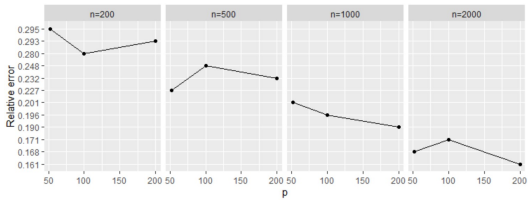
(h) $\rho = 0.5$, PRPRVM-POET



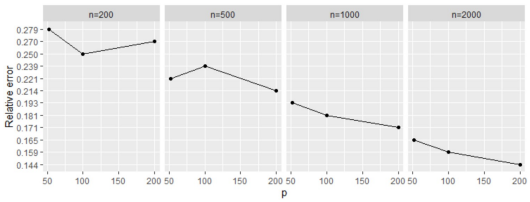
(i) $\rho = 0.5$, PRVM



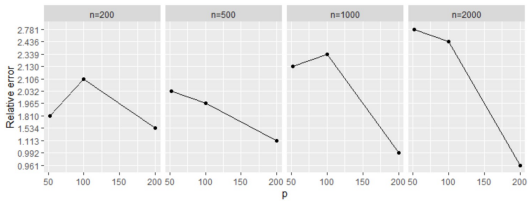
(j) $\rho = 0.5$, PPRVM



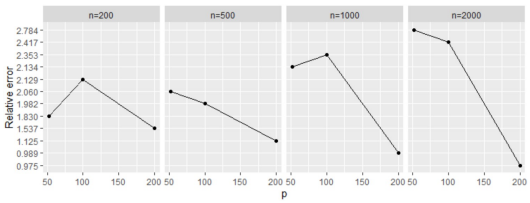
(k) $\rho = 0.5$, CRPRVM-POET



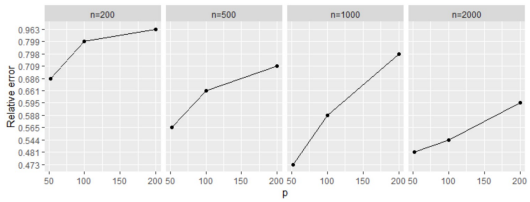
(l) $\rho = 0.5$, PCRPRVM-POET



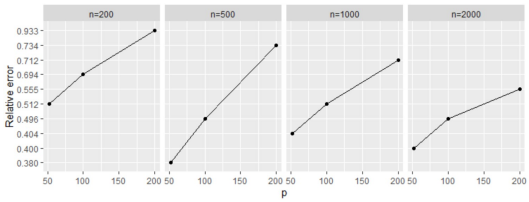
(m) $\rho = 0.9$, RPRVM-POET



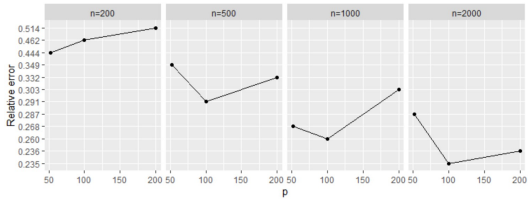
(n) $\rho = 0.9$, PRPRVM-POET



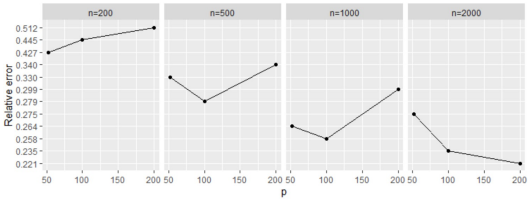
(o) $\rho = 0.9$, PRVM



(p) $\rho = 0.9$, PPRVM



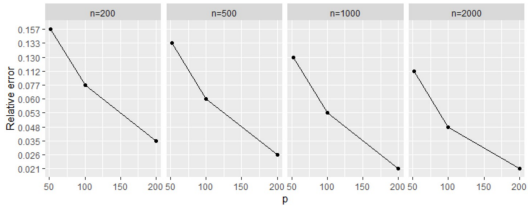
(q) $\rho = 0.9$, CRPRVM-POET



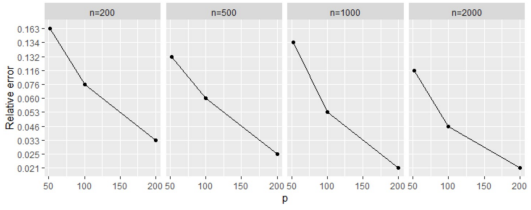
(r) $\rho = 0.9$, PCRPRVM-POET

Figure D.1 Average relative errors against the dimensionality p under the exponentially decaying cross-sectional correlation with $\rho \in \{0, 0.5, 0.9\}$.

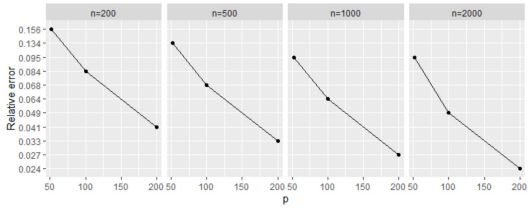
D.2 $|\omega^{\mathrm{T}}(\widehat{\Gamma} - \Gamma)\omega|$ against the dimensionality p



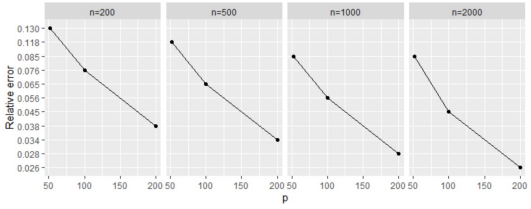
(s) $\rho = 0.5$, RPRVM-POET



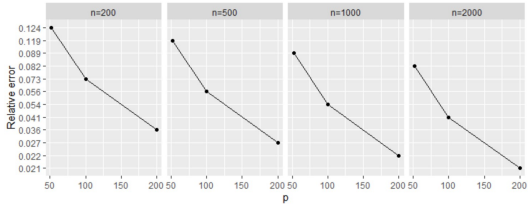
(t) $\rho = 0.5$, PRPRVM-POET



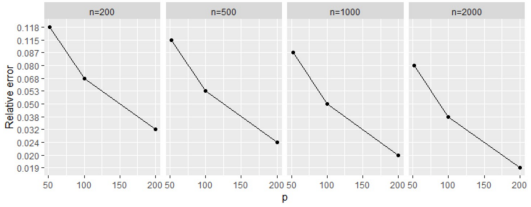
(u) $\rho = 0.5$, PRVM



(v) $\rho = 0.5$, PPRVM



(w) $\rho = 0.5$, CRPRVM-POET



(x) $\rho = 0.5$, PCRPRVM-POET

Figure D.2 $|\omega^{\mathrm{T}}(\widehat{\Gamma} - \Gamma)\omega|$ against the dimensionality p under the exponentially decaying cross-sectional correlation with $\rho = 0.5$.